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Modelling and updating of large surface-to-surface joints in the AWE-MACE structure

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Abstract

Model updating of joints in the AWE-MACE system is carried out using a sensitivity method. The joints are characterised by large surface-to-surface contact regions and are excited in vibration tests within the linear range. The joints are modelled using a layer of special interface elements having material properties that may be adjusted to improve the prediction of the complete model. A series of three updating exercises are described and it is shown that by using only six parameters based upon the circumferential-wave and bending modes that the prediction of the axial and torsional modes is improved sufficiently to be of practical usefulness for many applications. Fewer numbers of updating parameters are found to be sufficient to correct different subsets of vibration modes. Linear equivalent models identified by this approach are found to be valid within the usual range of vibration tests.

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Keywords: Joints; Finite element model updating; Validation

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1. Introduction

The urgent need for a solution to the problem of modelling complicated multi-component structures has been recognised by both UK and US government agencies in recent reports [1,2]. The finite element representation of joints between different components is particularly in need of attention. Of course, joints between components are many and varied and it appears that most previous studies have concentrated upon joints with physical dimensions that are small when compared to the overall dimensions of the complete assembly. Probably the bolted joint between linkages or flanges is the most researched [3–9], although spot-welded joints between thin plates (as in motor car body shells) have also been studied in considerable detail [10–14]. The behaviour of joints under dynamic loads is especially interesting because the rubbing of surfaces gives rise to energy dissipation by friction. Friction is of course a non-smooth non-linear phenomenon under oscillatory motion and when the relative motion across a joint becomes large there may be other sources of non-linearity, such as stops, clearances and changing contact areas that need to be taken into account.

In the present study the problem of large contact interfaces is addressed. The Modal Analysis Correlation Exercise (MACE) structure, the subject of this research, is an unclassified structure from AWE-Aldermaston UK. The complete MACE structure contains many different types of mechanical joints described by Reece [15]. The work presented in this article, however describes the application of the finite element model updating method [16,17] to joints between three components, the CASE, BODY and COLLAR. The work is limited to small-amplitude vibrations thereby allowing the identification of linearised joint models. The results reported mark the completion of the first phase of a research programme, which includes planned further investigations into different joint types and the removal of the present restriction to small joint displacements.

2. MACE components, joints and finite element model

The CASE, BODY and COLLAR of MACE, shown separately in Fig. 1 and when assembled in Fig. 2, are manufactured from aluminium. All three are axisymmetric except that the CASE has

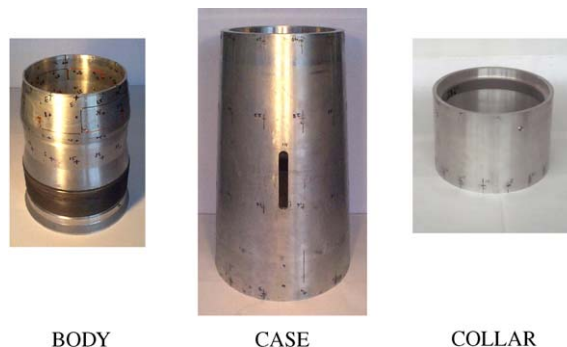


Fig. 1. MACE components.

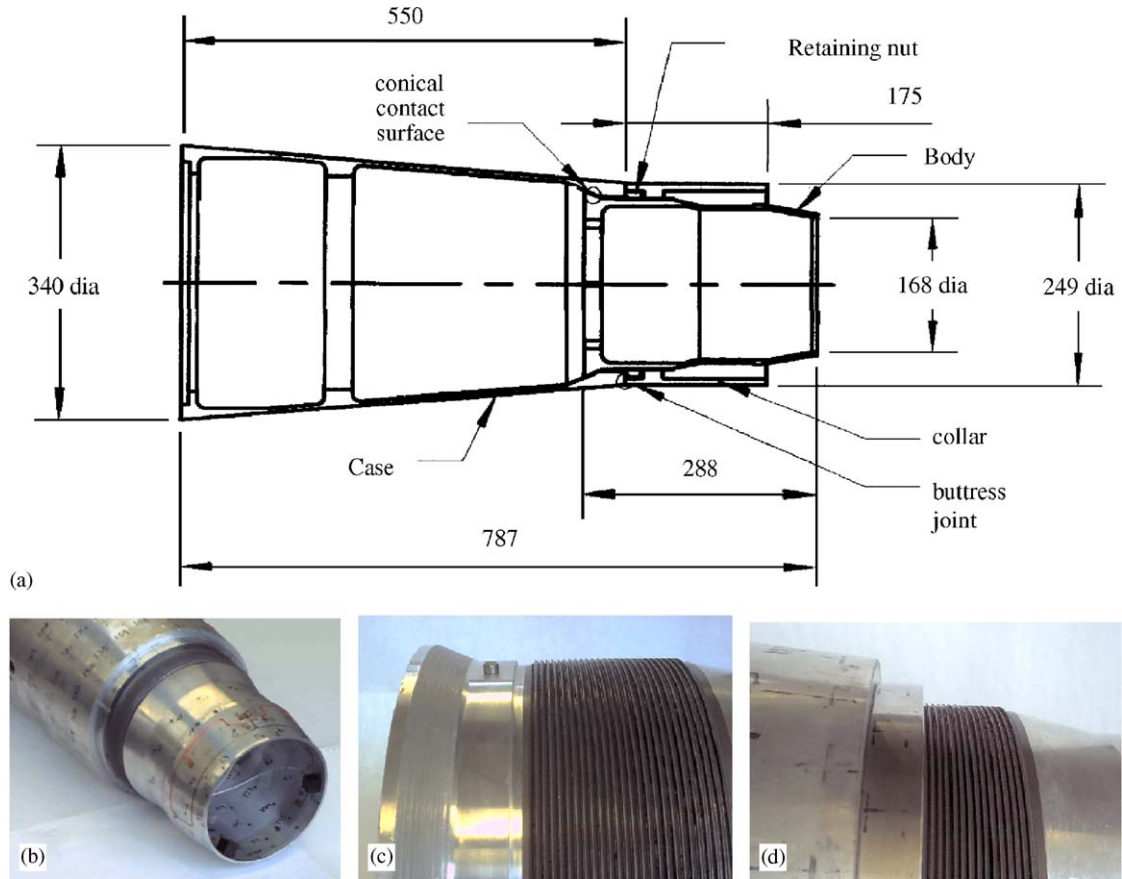


Fig. 2. (a) Assembled MACE, (b) BODY and CASE, (c) Conical contact surface at the base of the BODY, (d) BODY, CASE and retaining ring.

three equally spaced narrow slots in the axial direction. When the BODY is inserted into the CASE it is initially held in place by a thin retaining ring, which brings the interior conical surface close to the top of the CASE into contact with the exterior conical surface at the base of the BODY. When the contact between these two has been firmly established the retaining ring is slackened off slightly and the COLLAR passed over the head of the BODY and tightened up on the same screw threads, thereby holding all three components in place and making a further joint, the abutment between the base of the COLLAR and the head of the CASE. There are therefore three joints: the conical-surface joint between the BODY and CASE, the screw-threaded joint between the COLLAR and BODY, and the abutment between the CASE and COLLAR. The retaining nut, which would normally add a small amount of mass to the system but virtually no stiffness, was omitted in all the tests described in this paper. In the final assembly all three joints are loaded by internal forces, and the stiffness of the joints depends on the torque applied to the COLLAR at assembly. The overall dimensions of MACE are 787 mm long by 340 mm maximum diameter, and the diameter across the joints is

almost 249 mm. Thus it can be seen that the size of the joints are of the same order as the overall dimensions.

The finite element model, shown in Fig. 3, was created in MSC-NASTRAN using 4-noded plate elements (CQUAD4) and 8-noded solid elements (CHEXA) and the models of the three separate components were each validated by modal tests. The results are shown in Table 1.

It can be seen that the natural frequencies appear in close pairs (exact pairs in the finite element model) except for the fifth and sixth modes in the CASE. The close natural frequencies arise because the nodes of mode shapes in axisymmetric structures do not have any preferred location. In the CASE, however, the three axial slots cause the nodes to be fixed when the mode shape has three circumferential waves. In the lower mode (FE: 1194 Hz) the vibration nodes coincide with the slots, and in the higher mode (FE: 1244 Hz) the antinodes are at the slots. The closeness of the finite element predictions to the natural frequencies found in the modal tests shows that separate components are well modelled and that any significant errors in finite element results for the assembled MACE will be due to mis-modelling of the contact interfaces between the components.

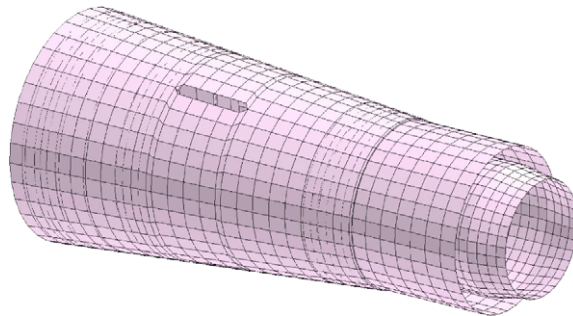


Fig. 3. Finite element model.

Table 1
Measured and predicted natural frequencies (Hz)

Mode no.	CASE		BODY		COLLAR	
	Test	FE	Test	FE	Test	FE
1	586	592	536	538	512	526
2	586	592	537	538	514	526
3	707	725	1169	1151	825	830
4	713	725	1175	1151	826	830
5	1186	1194	2005	1956	1299	1314
6	1252	1244	2013	1956	1300	1314
7	1308	1317	2056	2042	2171	2211
8	1330	1317	2065	2042	2173	2211

2.1. Joint contact interfaces

Modelling of the interfaces between the three components was of course an issue of special concern. In the literature, two methods predominate for the modelling of contact interfaces: (i) zero-thickness elements with special constitutive equations [18–20] and (ii) a thin layer of elements [21–23] with different properties from the surrounding material at the contacting surfaces of each of the mating components. In the first of these methods the contact stiffnesses are usually considered to remain constant over a range of interface displacements until the joint yields (when the friction capacity is exceeded). For joints that behave linear-elastically in the closed condition the constitutive relationship can be expressed as

$$\begin{aligned}\Delta\sigma &= k_n\Delta v, \\ \Delta\tau_1 &= k_s\Delta u_1, \\ \Delta\tau_2 &= k_s\Delta u_2,\end{aligned}\tag{1}$$

where $\Delta\sigma$, $\Delta\tau_1$ and $\Delta\tau_2$ are, respectively, the elastic part of the incremental normal and tangential stresses and Δv , Δu_1 and Δu_2 are the incremental relative normal and tangential displacements across the joint. The subscripts ‘1’ and ‘2’ denote the two orthogonal tangential displacement directions in the contact plane. The parameters k_s and k_n are numbers which penalise surface penetration and slipping, respectively. Once the joint exceeds its elastic limit, sliding follows an elastic-plastic law. When in a state of sliding a quasi-linear constitutive equation at each increment can be defined [24] as

$$\begin{Bmatrix} \Delta\sigma \\ \Delta\tau_1 \\ \Delta\tau_2 \end{Bmatrix} = \begin{bmatrix} k_n(1 - \mu^2 k_n^2/H) & -k_n k_s \beta_1 \mu/H & -k_n k_s \beta_2 \mu/H \\ & k_s(1 - \beta_1^2 k_s/H) & -k_s^2 \beta_1 \beta_2/H \\ & \text{sym} & k_s(1 - \beta_2^2 k_s/H) \end{bmatrix} \begin{Bmatrix} \Delta v \\ \Delta u_1 \\ \Delta u_2 \end{Bmatrix},\tag{2}$$

where

$$H = k_n \mu^2 + k_s, \quad \beta_1 = \tau_1 / \sqrt{\tau_1^2 + \tau_2^2}, \quad \beta_2 = \tau_2 / \sqrt{\tau_1^2 + \tau_2^2}, \quad \mu = \tan \varphi\tag{3}$$

and φ is the friction angle.

The validity of the model developed using the second method is restricted to elastic (or quasi-elastic) closed-state joints. This means that either there is no slippage across the joints and the forces applied are less than the limit defined by friction and adhesion, or if there is slippage it is small and can be represented adequately by a linear model. The thin layer of interface elements in the present model has isotropic material properties, though in principle it is possible to assign anisotropic material properties, which would allow for very considerable adjustment of the joint behaviour including coupling effects between normal and shear stiffness terms if this could be physically justified.

The linear constitutive equation for CHEXA elements, which form the interface layer, may be written generally in the form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ & & c_{33} & c_{34} & c_{35} & c_{36} \\ & & & c_{44} & c_{45} & c_{46} \\ & & & & c_{55} & c_{56} \\ \text{sym} & & & & & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}. \tag{4}$$

Then, assuming that at each joint the properties of the interface layer remain constant from one element to the next, there are 21 parameters for each joint, i.e. c_{ij} , $i = 1, \dots, 6, j = i, \dots, 6$, to be identified. One way to reduce the parameters to an acceptable number is by applying various physical constraints as was done for example by the present authors [25] to develop a plate element with minimal discretisation error and again [26] in model updating to locate a crack in a beam.

Both approaches result in equivalent models. In the first case the coefficients of distributed contact stiffnesses must be estimated and in the second the material properties of the contacting elements will depend upon the thickness of the thin layer. For the present study the second approach was adopted with the thin layers assumed to be composed of isotropic materials. A section through the finite element model showing all three joints is provided in Fig. 4. The interface elements are shown shaded. It can be seen that the CHEXA elements are used in the region of the joints whereas CQUAD4 elements are used elsewhere.

2.2. Updating procedure and design parameters

Model updating was carried out using the *Design Sensitivity* module available in MSC-NASTRAN Solution 200, the first eight natural frequencies being included in the objective function

$$\min \sum_{i=1}^n W_i (1 - \omega_i^e / \omega_i^a)^2 \tag{5}$$

and the weights set to unity, $W_i = 1, i = 1, \dots, 8$. The permissible variation of the design parameters was within the range 0.0001–1.1 of their initial values, thereby allowing a substantial

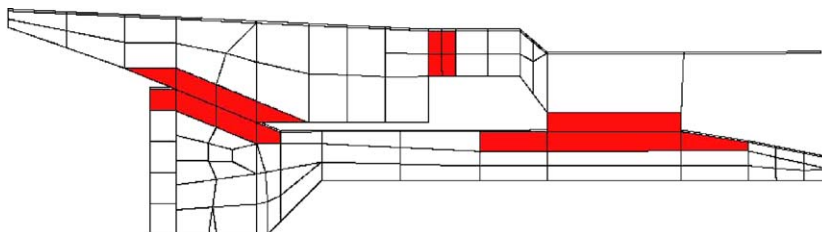


Fig. 4. Interface elements.

reduction in the interface stiffnesses, but preventing any unrealistic increases of stiffness across the joints. The selected design parameters were the elastic and shear moduli of the joints, with initial values determined from $E_{\text{aluminium}} = 72 \text{ GN/m}^2$ and $\nu = 0.3$. Three separate updating exercises were in fact carried out. In two of them only the elastic modulus was corrected and in the third and final exercise, six parameters were corrected, namely E and G for each of the three joints.

3. Vibration tests

In the first vibration test MACE was assembled and free–free conditions for the test were approximated by a suspension comprising soft springs attached to the CASE at the bottom of the skirt. It was difficult to attach shakers to the COLLAR or BODY because of the curved surfaces. However, small threaded holes at the skirt of the CASE were available and these were used to attach two shakers (LING 401) with axes along different CASE radii. This arrangement had the disadvantage that energy had to be transmitted through the joints to excite the BODY and COLLAR, both of which were active in all the lower modes of the assembled system. A wire frame consisting of 89 (roving) accelerometer points (48 on the CASE, 5 on the BODY and 36 on the COLLAR) was set up and frequency responses were estimated using a multiple-input, multiple-output (MIMO) routine. One disadvantage was that the bending modes were not strongly excited. However, good estimates of the natural frequencies and modes shapes of the circumferential-wave modes were obtained and used to carry out the first updating exercise.

In a second test a specially modified COLLAR was manufactured with a closed end that fully enclosed the BODY when assembled. This allowed the attachment of a reaction mass of 10.8 kg to the collar having the effect of reducing the natural frequencies of the bending modes and making them easier to excite. The replacement COLLAR introduced a further mass of 5.7 kg in addition to the mass of the original one. The special COLLAR and the new assembly with the added mass

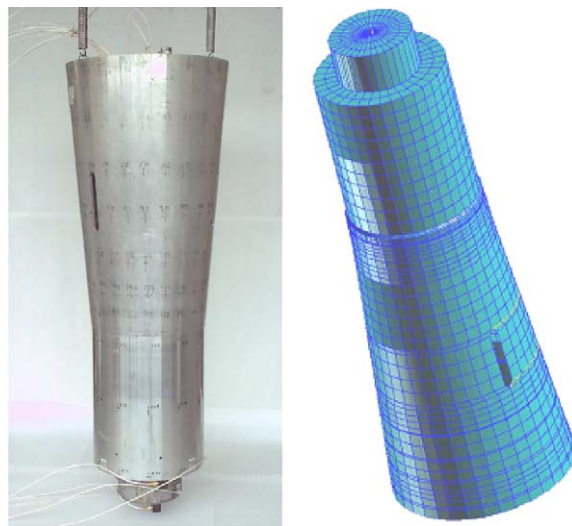


Fig. 5. MACE with modified COLLAR and added mass.

are shown in Fig. 5. A roving hammer test was carried out using seven uniaxial fixed accelerometers and the same 89-point wire frame as before. A MIMO frequency response estimator was used. This was followed by a second round of model updating in two exercises with different numbers of parameters. Finally, separate tests were carried out to determine the axial and torsional modes, which were used to validate the updated model of the joints. Fig. 5(a) shows the arrangement of the experiment for the torsional test. The stud at the top of the picture, on the skirt, was used for tangential excitation by a hammer. There were three triaxial accelerometers on both the COLLAR and the mass (one of each can be seen in the figure). A further nine triaxial accelerometers were attached to the CASE and three to the BODY making a wire frame of six equilateral triangles along the length of the assembled structure.

4. Results and discussion

A preliminary model updating exercise using only two design parameters was carried out using vibration test data from the assembled MACE with the original COLLAR. Data from a second test with the modified COLLAR and added mass was used in two updating exercises, the first with three parameters and the second with six. The final updated model was validated using independently measured natural frequencies for the first axial and first torsional modes.

4.1. First vibration test—first updating exercise

In the first vibration test only the circumferential-wave modes were excited, and as might be expected, sensitivity studies revealed that the shear stiffness across the joints had virtually no effect on the natural frequencies in the range of the first eight modes. The normal stiffnesses of the BODY–COLLAR threaded joint and the COLLAR–CASE buttress joint were found to be significantly more important parameters than the normal stiffness of the BODY–COLLAR conical surface joint, presumably because the latter joint is stiffened by the base of the BODY and the greater wall thickness of the CASE close to the conical joint. Therefore just two parameters, the elastic moduli of the interface elements, were used to correct the BODY–COLLAR and COLLAR–CASE joints. The measured natural frequencies, together with the initial predictions and updated results from the finite element model, are shown in Table 2.

Table 2
Table of natural frequencies—first updating exercise

Mode no.	Test (Hz)	Initial FE (Hz)	Updated FE (Hz)	Initial FE % error	Updated FE % error
1	602	618	615	2.66	2.16
2	602	618	615	2.66	2.16
3	836	967	884	15.67	5.89
4	836	967	884	15.67	5.89
5	1174	1183	1156	0.77	–1.53
6	1174	1183	1156	0.77	–1.53
7	1207	1245	1235	3.15	2.32
8	1308	1306	1282	–0.15	–1.99

The seventh and eighth modes are the three-circumferential-wave modes separated by the slots in the CASE. The initial finite element model is in error in its predictions by over 15% in the third and fourth modes and it is remarkable that this error can be reduced to less than 6% by adjusting only two parameters. The updated model had converged fully in eleven iterations the values of the design parameters (E —COLLAR/BODY and E —COLLAR/CASE) having been reduced to 0.01 and 0.014 of the elastic modulus of the parent material.

4.2. Second vibration test—second and third updating exercises

Three parameters, the elastic moduli of the interface-layer elements in each of the three joints, were used in the first of the two updating exercises carried out using data from the second test. Poisson's ratio was maintained constant at $\nu = 0.3$. In the second exercise using second vibration test data six parameters were updated, the elastic and shear moduli at each joint. The results of the two updating exercises are summarised in Table 3. The pairs of circumferential-wave modes that appear with identical natural frequencies (in the finite element model) are shown as a single entry in the table and not as two separate frequencies (as in the previous Table 2). It is of course important to remember that the measured first torsional and axial modes were not used in correcting the model and the error shown in the table is therefore an indicator of the validity of the updated model in predicting not only the circumferential-wave and bending modes, but also the torsional and axial ones.

The errors in the initial finite element model are very significant, as high as 41%, but after updating the highest error is less than 6%. Discounting the torsional and axial modes it can be seen that the results from the second and third updating exercises appear to be very similar. The convergence plots for the elastic moduli, shown in Fig. 6(a) and (b), starting from the material properties of the parent material, also seem to be similar. It may be seen from Fig. 6(b) and (c) that the elastic and shear moduli for the CASE–BODY and CASE–COLLAR joints converge similarly with the same constant of proportionality, but do not for the joint between the BODY and COLLAR. This accounts for the improved estimate of the torsional mode when using six

Table 3
Natural frequencies: second and third updating exercises

Measured		Initial FEM		Second updating exercise 3-parameters		Third updating exercise 6-parameters	
Mode no.	Hz	Hz	% error	Hz	% error	Hz	% error
1	551	777	41	549	−0.36	546	−0.9
2	612	635	3.75	622	1.63	622	1.63
Tors'l	1055	1161	9.13	917	−13.08	1018	−3.5
Ax'l	1021	1285	25.86	1076	5.39	1079	5.7
3	1119	1186	5.98	1111	−0.71	1125	0.53
4	1175	1163	−1.02	1166	−0.76	1177	−0.17
5	1337	1415	5.83	1330	−0.52	1334	−0.22
6	1516	1643	8.37	1604	5.8	1604	5.8
7	1645	1848	12.34	1679	2.06	1687	2.55
8	1717	1761	2.56	1742	1.45	1744	1.57

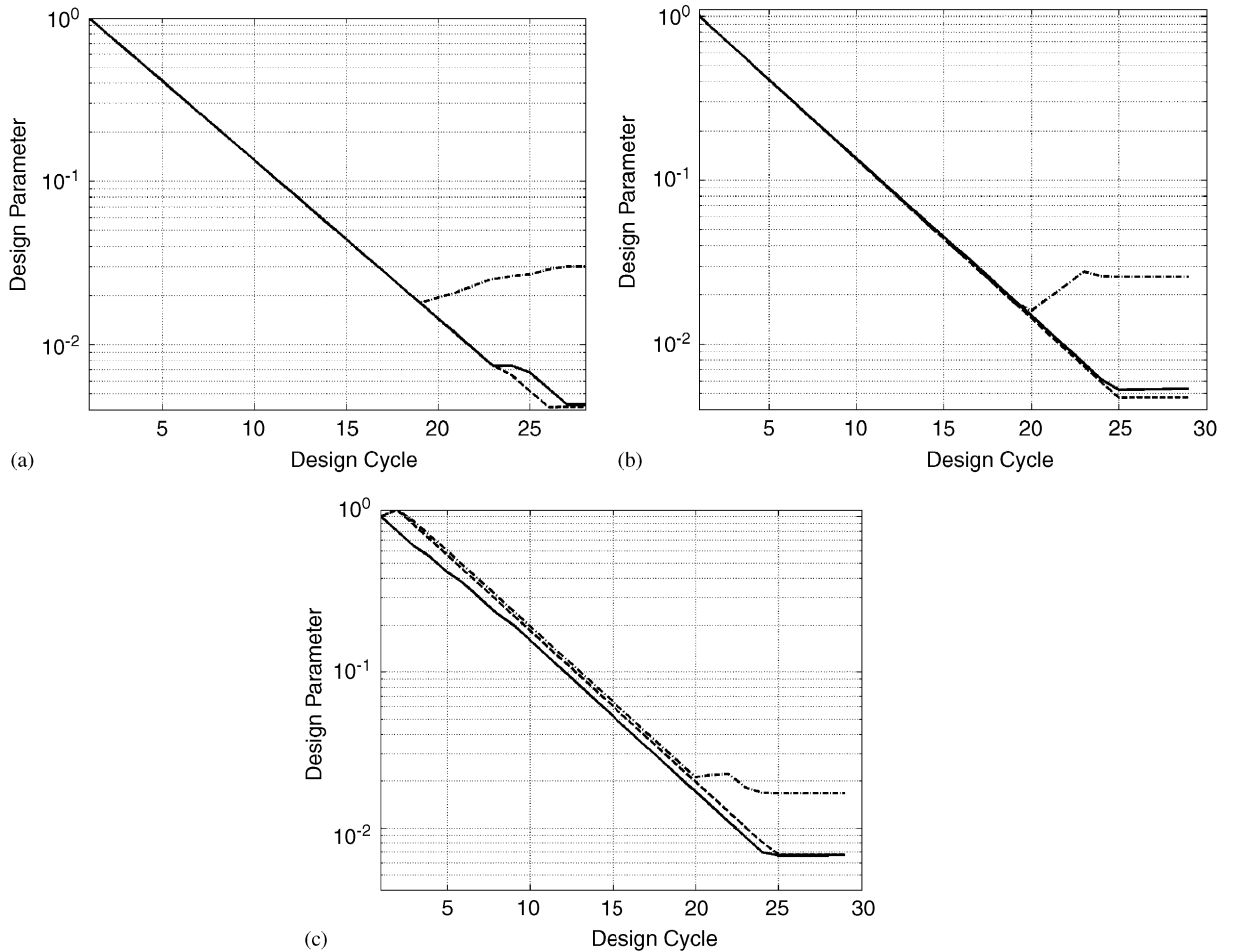


Fig. 6. Convergence plots—(a) *E*: 3 parameters, (b) *E*: 6 parameters and (c) *G*: 6 parameters. (Full line: CASE-BODY, Dashed: CASE-COLLAR, Dash-Dot: BODY-COLLAR).

Table 4
Table of non-dimensionalised sensitivities

Mode	<i>E</i>			<i>G</i>		
	CASE-BODY	CASE-COLLAR	COLLAR-BODY	CASE-BODY	CASE-COLLAR	COLLAR-BODY
1	9.61×10^{-2}	3.62×10^{-1}	1.11×10^{-1}	7.86×10^{-2}	-7.82×10^{-2}	-1.59×10^{-2}
2	1.15×10^{-1}	6.28	4.88×10^{-1}	1.89×10^{-1}	-3.64	-1.47×10^{-1}
3	9.05×10^{-1}	8.84×10^{-1}	2.05	8.24×10^{-1}	-1.41×10^{-1}	-8.86×10^{-1}
4	1.59	1.52	1.3×10^{-1}	-2.53×10^{-1}	5.58×10^{-1}	5.33
5	2.11	1.53	6.82×10^{-2}	6.46×10^{-1}	2.88×10^{-1}	4.32×10^{-1}
6	1.99	8.32×10^{-1}	5.07×10^{-2}	1.08×10^{-1}	1.29×10^{-1}	1.17×10^{-1}
7	6.24×10^{-1}	2.76×10^{-1}	7.43×10^{-2}	2.07×10^{-1}	1.39×10^{-1}	1.03×10^{-1}
8	2.49×10^{-1}	3.47×10^{-1}	2.07×10^{-1}	1.05	1.08	1.91×10^{-1}

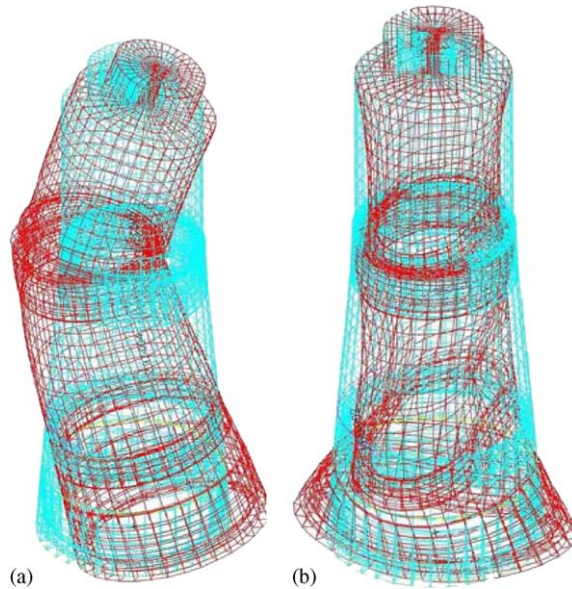


Fig. 7. (a) Mode 1 and (b) Mode 4.

parameters because mode 4 is sensitive to the shear modulus of the COLLAR–BODY joint as shown in Table 4. It is seen from the table that the shear modulus is sensitive at the different joints for several of the circumferential-wave modes. Thus, when six parameters were updated (on the basis of the measured circumferential-wave and bending modes) it was found that the torsional and axial modes were improved sufficiently well for the model to be considered valid for many applications within the frequency range of the tests. The first mode, the one in greatest error in the initial finite element model, is a bending mode as shown in Fig. 7(a). Mode 4 is presented in Fig. 7(b), which shows how shearing can take place across the joints.

5. Conclusions

The AWE MACE structure is a complicated multi-component system with several surface-to-surface joints. Model updating has been carried out to determine equivalent models of the joints between three components, the CASE, BODY and COLLAR within the range of linear, small vibrations. It was found to be necessary to update only two parameters for the correction of the circumferential-wave modes. For correction of the bending modes as well, three updating parameters were needed. When six parameters (the elastic and shear moduli of the three joint) were updated based upon measured circumferential-wave and bending modes, it was found that the prediction of the torsional and axial modes were sufficiently improved that the model could be regarded as valid for many applications. Updating was found to be a valid approach for the estimation of complicated large area joints within systems such as MACE.

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