باسمه تعالی

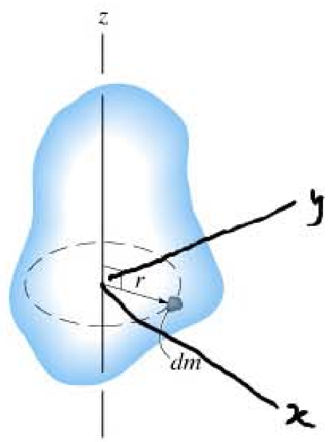
۸۷/۳/۱۲ و ۸۷/۳/۷

گشتاور لختی جرم : معیار تفاوت جسم نسبت به شتاب زاویه‌ای حول محور مورد نظر

$$I = \int r^2 dm > 0$$

$kg \cdot m^2$

$slug \cdot ft^2$

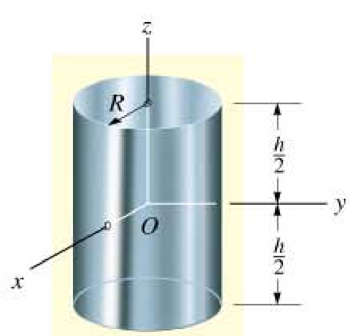


$$I_z = \int r^2 dm = \int (x^2+y^2)\, dm$$

$$I_x = \int (z^2+y^2)\, dm$$

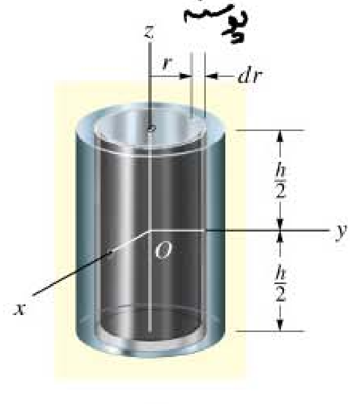
$$I_y = \int (x^2+z^2)\, dm$$

$$I = \int r^2 dm = \int r^2 \rho\, dv = \rho \int r^2 dv$$



(a)

پوسته



(b)

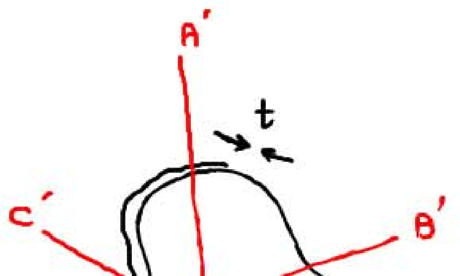
$$I_z = \int r^2 dm$$

$$dI_z = r^2 dm = 2\pi h r^3 dr$$

$$dm = \rho 2\pi r h\, dr$$

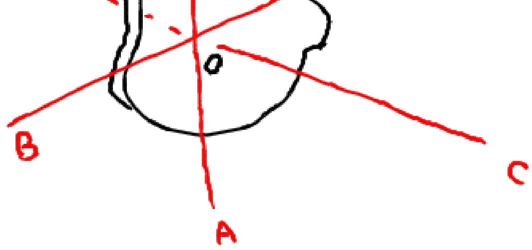
$$I_z = \rho \int_0^R 2\pi h r^3 dr = \rho \frac{2\pi h R^4}{4} = \rho \frac{\pi h R^4}{2} = \frac{mR^2}{2}$$

$$m = \pi R^2 h \rho$$



گشتاور لختی ورقهای نازک

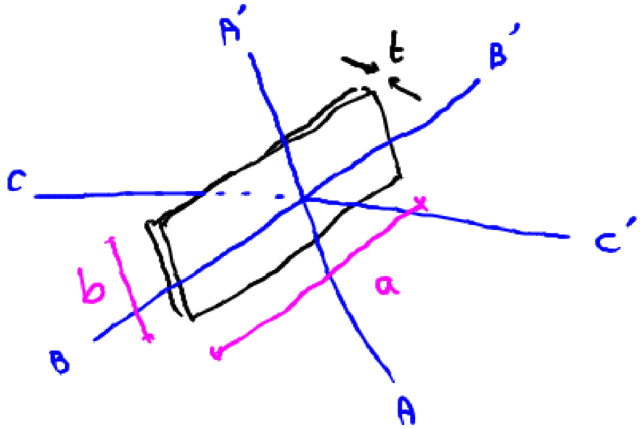
$$I_{AA',G} = \int r^2 dm = \rho t \int r^2 dA = \rho t\, I_{AA',A}$$



$$dm = \rho t \, dA$$

$$I_{cc',G} = \rho t \, J_{o,A}$$

مثال:

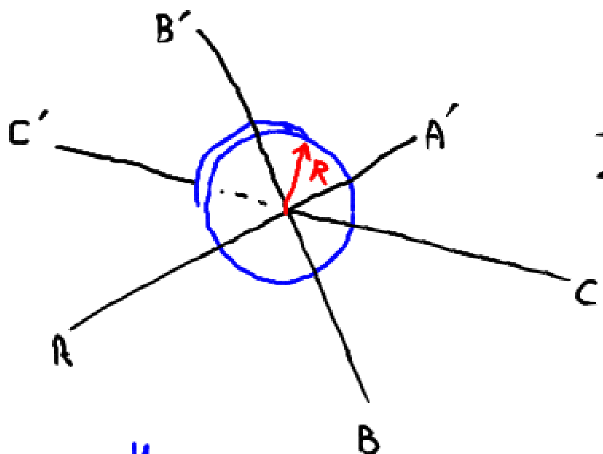


$$I_{AA',G} = \rho t \frac{a^3 b}{12} = \frac{m a^2}{12}$$

$$I_{BB',G} = \rho t \frac{a b^3}{12} = \frac{m b^2}{12}$$

$$I_{cc',G} = \rho t \left(\frac{a^3 b}{12} + \frac{a b^3}{12} \right) = \frac{m}{12}(a^2 + b^2)$$

$$m = \rho a b t$$

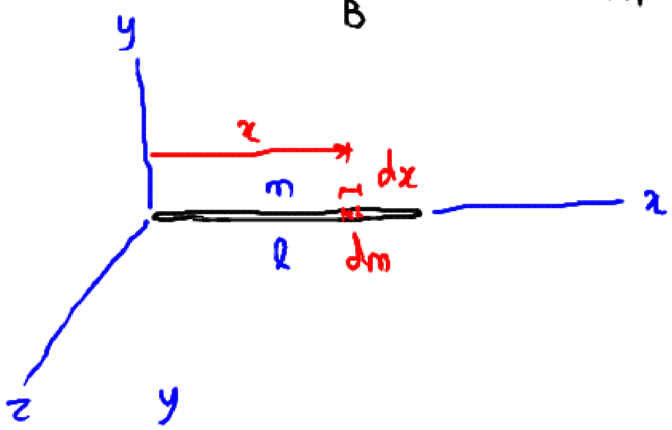


$$I_{BB',G} = I_{AA',G} = \rho t \frac{\pi R^4}{4} = \frac{m R^2}{4}$$

$$I_{cc',G} = \rho t \frac{\pi R^4}{2} = \frac{m R^2}{2}$$

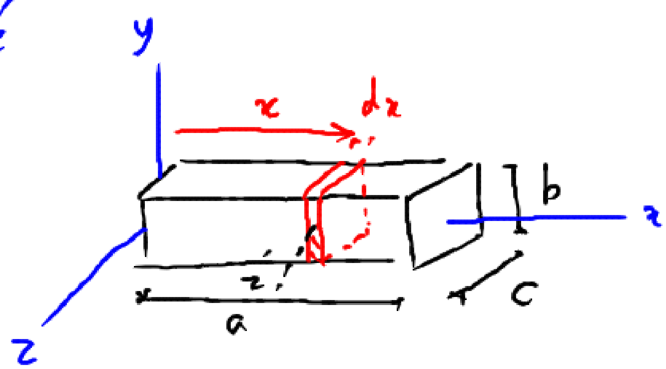
$$m = \pi R^2 t \rho$$

مثال:



$$I_{y,G} = \int r^2 dm = \int x^2 dm = \int_0^l \frac{m x^2}{l} dx$$

$$dm = \frac{m}{l} dx \qquad I_{y,G} = \frac{m l^2}{3}$$



$$I_{z,G} = \int r^2 dm$$

$$dm = \rho \, b c \, dx$$

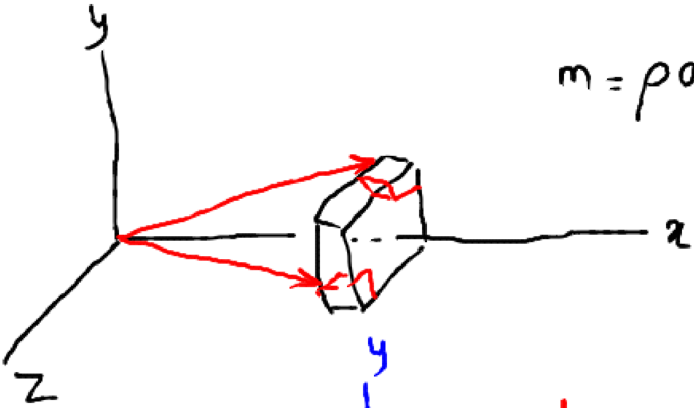
$$dI_{z'} = \frac{b^2}{12} dm$$

قضیه‌ی محورهای موازی $\quad dI_z = dI_{z'} + dm \, d^2$

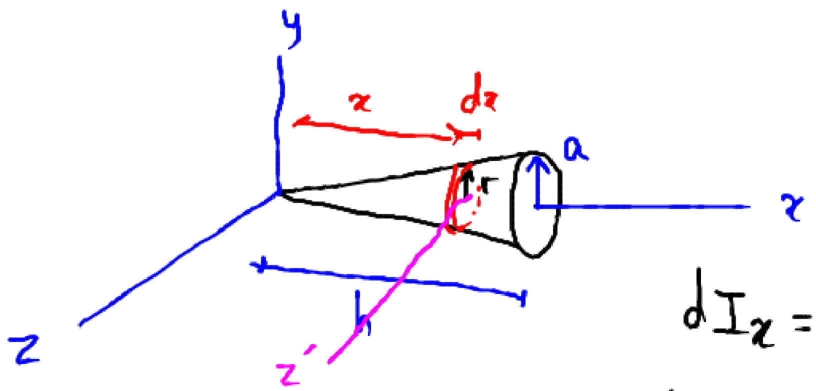
$$dI_z = \frac{b^2}{12}dm + dm\,x^2 = \left(\frac{b^2}{12}+x^2\right)\rho bc\,dx$$

$$I_z = \int_0^a \left(\frac{b^2}{12}+x^2\right)\rho bc\,dx = \rho bc\left(\frac{ab^2}{12}+\frac{a^3}{3}\right) = m\left(\frac{b^2}{12}+\frac{a^2}{3}\right)$$

$$m = \rho abc$$

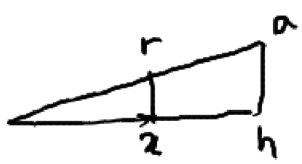


مثال:



$$dI_x = \frac{dm\,r^2}{2}$$

$$dm = \rho\pi r^2 dx$$



$$\frac{r}{a} = \frac{x}{h} \longrightarrow r = \frac{a}{h}x$$

$$dI_x = \frac{1}{2}\rho\pi r^4 dx = \frac{1}{2}\rho\pi\left(\frac{a}{h}x\right)^4 dx$$

$$I_x = \int_0^h \frac{1}{2}\rho\pi\frac{a^4}{h^4}x^4 dx = \frac{1}{2}\rho\pi\frac{a^4}{h^4}\frac{h^5}{5} = \frac{1}{10}\rho\pi a^4 h$$

$$m = \rho\frac{1}{3}\pi a^2 h$$

$$I_x = \frac{3}{10}ma^2$$

$$dI_{z'} = \frac{r^2 dm}{4} = \frac{r^2}{4}\rho\pi r^2 dx = \rho\pi\frac{r^4}{4}dx = \frac{\rho\pi}{4}\left(\frac{a}{h}x\right)^4 dx$$

قضیه محور موازی $dI_z = dI_{z'} + x^2 dm$

$$dI_z = \frac{\rho\pi}{4}\left(\frac{a}{h}x\right)^4 dx + x^2\rho\pi\left(\frac{a}{h}x\right)^2 dx$$

$$I_z = \rho\pi \frac{a^2}{h^2} \int_0^h \left(\frac{a^2}{4h^2} x^4 + x^4 \right) dx$$

$$I_z = \rho\pi \frac{a^2}{h^2} \left(\frac{a^2}{4h^2} + 1 \right) \frac{h^5}{5} = \frac{3}{5} m \left(\frac{a^2}{4} + h^2 \right)$$

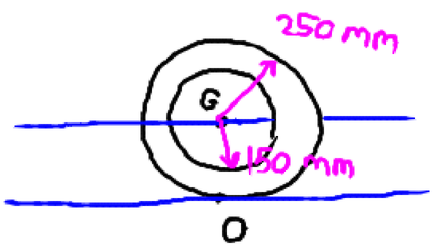
$$m = \rho \frac{1}{3} \pi a^2 h$$

شعاع چرخش جرمی $k = \sqrt{\frac{I}{m}}$ $\qquad I = k^2 m$

قضیه محورهای موازی $I = \bar{I} + md^2$

$$k^2 = \bar{k}^2 + d^2$$

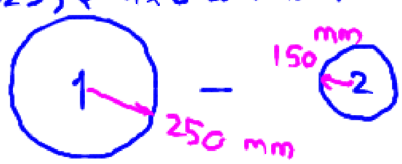
جرمهای مرکب: $\rho = 8000 \frac{kg}{m^3}$ $\quad t = 10$ mm



$I_G , I_O = ?$

$\frac{1}{2} m r^2$

$m_1 = \pi (.25)^2 \times .01 \times 8000 = 15.708$ kg



$m_2 = \pi (.15)^2 (.01) 8000$

$m_2 = 5.655$ kg

$I_{G_1} = \frac{1}{2} \times 15.708 \times .25^2 = .491 \ kg.m^2$

$I_{O_1} = I_{G_1} + m_1 (OG)^2 = .491 + 15.708 \times .25^2 = 1.473 \ kg.m^2$

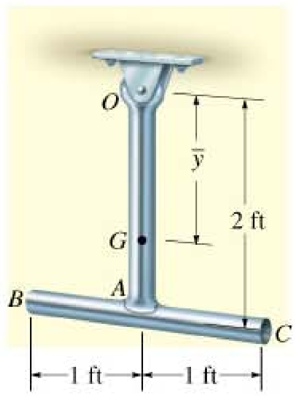
$I_{G_2} = \frac{1}{2} \times 5.655 \times .15^2 = .064 \ kg.m^2$

$I_{O_2} = .064 + 5.655 (.25)^2 = .417 \ kg.m^2$

$I_O = I_{O_1} - I_{O_2} = 1.473 - .417 = 1.056 \ kg.m^2$

$I_G = I_{G_1} - I_{G_2} = .491 - .064 = .427 \ kg.m^2$

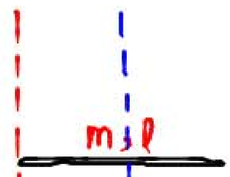
مثال: دو میله هرکدام به وزن 10 lb



I_O , I_G

مرکز جرم

$$m = \frac{10\ \text{lb}}{32.2\ \frac{\text{ft}}{\text{s}^2}} = .311\ \text{slug}$$



$$I = \frac{1}{3} m l^2$$

$$\bar{y} = \frac{\Sigma m_i \tilde{y}_i}{\Sigma m_i} = \frac{.311 \times 1 + .311 \times 2}{2 \times .311} = 1.5'$$

$$\bar{I} = I - md^2 = \frac{1}{3} m l^2 - m\left(\frac{l}{2}\right)^2$$

$$\bar{I} = \frac{ml^2}{12}$$