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OPTIMUM DESIGN OF STRUCTURES USING AN IMPROVED FIREFLY ALGORITHM

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ABSTRACT

Nature-inspired search algorithms have proved to be successful in solving real-world optimization problems. Firefly algorithm is a novel meta-heuristic algorithm which simulates the natural behavior of fireflies. In the present study, optimum design of truss structures with both sizing and geometry design variables is carried out using the firefly algorithm. Additionally, to improve the efficiency of the algorithm, modifications in the movement stage of artificial fireflies are proposed. In order to evaluate the performance of the proposed algorithm, optimum designs found are compared to the previously reported designs in the literature. Numerical results indicate the efficiency and robustness of the proposed approach.

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KEY WORDS: design optimization; truss structures; sizing optimization; geometry optimization; firefly algorithm; nature-inspired algorithms

1. INTRODUCTION

Since truss structures are widely used for structural applications, optimum design of this type of structures has a great importance. Generally, in design optimization of truss structures, the objective is to find the best feasible structure with a minimum weight. In other words, optimum design of truss structures is a search for the best possible arrangements of design variables according to the determined constrains. Design variables involved in optimum design of truss structures can be considered as sizing, geometry, and topology variables. In

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sizing optimization of truss structures, the aim is to find the optimum values for cross sectional areas of the elements. Geometry optimization means to determine the optimum positions of the nodes while presence or absence of the members are considered in the topology optimization.

Meta-heuristic approaches such as genetic algorithms [1], simulated annealing [2], particle swarm optimization [3], ant colony optimization [4, 5], harmony search method [6] etc., have been widely employed by researchers for solving optimization problems so far. These algorithms do not require gradients of objective functions, can deal with both discrete and continuous variables, and are able to handle both discrete and continuous variables. Such features are some reasons of popularity of meta-heuristic algorithms.

The firefly algorithm, proposed by Yang [7, 8], is a novel meta-heuristic approach which simulates the natural behavior of fireflies. In [8, 9] the superiority of firefly algorithm based approaches to both PSO and GA is demonstrated using various test functions. Additionally, satisfactory application of firefly algorithm to solving nonlinear design problems is reported in [10]. In [10] the firefly algorithm is employed to solve a standard pressure vessel design optimization problem. Recently, a discrete firefly algorithm with local search has been proposed for solving permutation flow shop scheduling problems [11].

In the present study, having assumed the topology of structures to be fixed, the authors carried out both sizing and geometry optimization of different types of truss structures using a modified firefly algorithm. The outline of the following sections of the paper is as follows: Section 2 contains an introduction to the firefly algorithm. In section 3, design optimization of truss structures using the firefly algorithm is described in detail. Section 4 presents the proposed modification in the movement stage of fireflies. In Section 5 the performance of the proposed algorithm is evaluated using typical design optimization problems of planar and spatial truss structures. Finally, section 6 includes the conclusion of the present study.

2. FIREFLY ALGORITHM

The firefly algorithm proposed by Yang [7, 8] is a recently developed search algorithm based on the natural behavior of fireflies. As described in [8], in order to develop the firefly algorithm, natural flashing characteristics of fireflies have been idealized using the following three rules:

1) All of the fireflies are unisex, therefore, one firefly will be attracted to other fireflies regardless of their sex.
2) Attractiveness of each firefly is proportional to its brightness, thus for any two flashing fireflies, the less bright firefly will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
3) The brightness of a firefly is determined according to the nature of the objective function.

The attractiveness of a firefly is determined by its brightness or light intensity which is obtained from the objective function of the optimization problem. However, the attractiveness β, which is related to the judgment of the beholder, varies with the distance between two fireflies. The attractiveness β can be defined by [10]:

$$\beta = \beta_o e^{-\gamma r^2}, \tag{1}$$

where r is the distance of two fireflies, β_o is the attractiveness at r = 0, and γ is the light absorption coefficient. The distance between two fireflies *i* and *j* at x_i and x_j, respectively, is determined using the following equation:

$$r_{ij} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2}, \tag{2}$$

where $x_{i,k}$ is the *k*-th parameter of the spatial coordinate x_i of the *i*-th firefly. In the firefly algorithm, the movement of a firefly *i* towards a more attractive (brighter) firefly *j* is determined by the following equation [10]:

$$x_i = x_i + \beta_o e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha \varepsilon_i, \tag{3}$$

where the second term is related to the attraction, while the third term is randomization with the vector of random variables ε_i using a normal distribution. More detailed descriptions of the firefly algorithm can be found in [7-10].

3. DESIGN OPTIMIZATION OF TRUSS STRUCTURES USING THE FIREFLY ALGORITHM

3.1 Problem formulation

Design optimization of truss structures can be formulated as follows [12]:

$$\text{Find } x = \{x_1, x_2, \ldots, x_d\}, \tag{4a}$$

$$x_{kl} \leq x_k \leq x_{ku}, \; k = 1, 2, \ldots, d \tag{4b}$$

to minimize

$$f(x) = W(x) + P(x) \tag{5}$$

subjected to

$$g_i(x) = \left| \frac{\sigma_i}{\sigma_{ai}} \right| - 1 \leq 0 \qquad i = 1, 2, \ldots, m \tag{6}$$

$$g_j(x) = \left| \frac{\delta_j}{\delta_{aj}} \right| - 1 \leq 0 \qquad j = 1, 2, \ldots, h, \tag{7}$$

where in equation (4a) and (4b), x is a candidate design (firefly), x_{kl} and x_{ku} are the lower and upper bounds of the *k*-th design variable x_k, and *d* is the total number of parameters of a firefly. In equation (5), f(x) is the objective function of the truss optimization problem, W(x) is the weight of the structure and P(x) is the penalty function. In equation (6) and (7), g_i and g_j are stress and displacement constraints, respectively, σ_i is the stress in the *i*-th member, σ_{ai} is the value of the allowable stress for the *i*-th member, δ_j is the displacement in the direction of the *j*-th degree of freedom and δ_{aj} is the allowable displacement in the same direction. Here, *m* is the number of truss members and *h* is the number of active degrees of freedom. Since the present optimization problem is a weight minimization problem, therefore, the brightness or light intensity of a firefly can be assumed to be the inverse of the corresponding objective function value.

3.2 Penalty function

In order to handle the predefined constraints of the design optimization problem, we used the following penalty function proposed by Rajeev and Krishnamoorthy [13]:

$$P(x) = W(x)KC, \tag{8a}$$

$$C = \sum_{r=1}^{s} g_r(x), \tag{8b}$$

where W(x) is the weight of the truss structure, K is a penalty constant, and g_r is the amount of violation of *r*-th constraint. In equation (8b), *s* is the total number of constrain evaluations for each firefly. In the present study an adaptive penalty function proposed in [14] is employed wherein K initiates from a minimum value in the beginning of the optimization process and then gets modified in each generation as follows:

$$K(t) = K(t\text{-}1) + \Delta K \qquad \text{if the best firefly is infeasible,} \tag{9a}$$

$$K(t) = K(t\text{-}1) - \Delta K/2 \qquad \text{if the best firefly is feasible,} \tag{9b}$$

where ΔK is the step size, and K(*t*) is the value of parameter K in the *t*-th generation. In this paper, the term *generation* is assumed to be equivalent to the number of structural analyses.

4. MODIFIED MOVEMENT STAGE

As mentioned before, in the firefly algorithm, the movement of a firefly *i* towards a brighter firefly *j* is determined by equation (3). Since x_j is brighter than x_i, we propose to update the position of firefly *i* based on the current position of firefly *j*. Therefore, instead of moving firefly *i* towards *j*, we propose searching the vicinity of firefly *j* which is a more reliable area. To do this, we replaced x_i by x_j equation (3) and used the following equation:

$$x_i = x_j + \beta_o e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha\varepsilon_i \tag{10}$$

In this study, a firefly *i* is compared to all members of the population in order to find brighter fireflies. In equation (10), ε_i is chosen using a normal distribution. Normal distribution has two parameters: a mean value and a standard deviation. In this study the mean value of the normal distribution is set to zero and the standard deviation is taken as the standard deviation of *k*-th parameter of all fireflies in each generation. This method of selecting the standard deviation of normal distribution can be found in [14]. In the present study, the parameters of fireflies which are not created within the bounds of design variables (sizing and geometry variables) are changed into the boundary values. Additionally, in case of discrete optimization, the values of discrete design variables (sizing variables) are changed into the values of nearest available sections. To avoid missing the brighter fireflies of the population, the position of a firefly is updated only if the new position found is better than the old one. Therefore, in the process of optimization each candidate design will be replaced only with a better design.

5. NUMERICAL EXAMPLES

5.1. Outline and parameter setting

In this section the performance of the proposed algorithm is evaluated using typical optimization examples of truss structures. For each example, the algorithm is executed 50 times and the best design found is reported. Optimum designs found, are compared to the previously reported results by other researchers. The general results of all 50 runs are given in Table 9.

For all examples studied in this section, a population of 50 fireflies is employed; the range of 0.5 to 1.5 is chosen for the penalty constant (K) with a step size (ΔK) of 0.1 [14]. The values of β_o and γ are both taken as 1 [10] and α is set to 0.5. The maximum number of structural analyses for examples 1 to 3 is 10000, and for the last example is 15000. Therefore, the algorithm terminates when the maximum number of structural analyses is met.

5.2. Example 1: Fifteen-bar truss structure

The sizing and geometry optimization of a 15-bar planar truss structure is performed in this example. The initial geometry of the truss is shown in Figure 1. A vertical load of 10 kips (44.48 kN) is applied at node 8. The stress limit is 25 ksi (172.369 MPa) in both tension and compression for all members. The material density is 0.1 lb/in.^{3}(2767.99 kg/m^3) and the modulus of elasticity is 10,000 ksi (68,947.6 MPa). For geometry optimization nodes 2, 3, 6 and 7, are allowed to move in both x and y directions; where nodes 6 and 7 have the same x coordinates as joints 2 and 3 respectively. Nodes 4 and 8 are permitted to move only in y direction. This example has totally 23 design variables including 15 sizing variables (cross-sectional areas of bars) and 8 geometry variables ($x_2 = x_6$, $x_3 = x_7$, y_2, y_3, y_4, y_6, y_7,

y_8).The available profile list for sizing variables is as follows: S = {0.111, 0.141, 0.174, 0.22, 0.27, 0.287, 0.347, 0.44, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18} in.^{2}. Table 2 gives the limits of geometry variables and Table 1 contains the results of optimization.

Table 1. Comparison of results for the fifteen-bar truss structure

Design variables	**Wu and Chow [15]**	**Tang et al. [16]**	**Hwang and He [17]**	**Rahami et al. [18]**	**The present work**
Sizing variables (in.^{2})					
A1	1.174	1.081	0.954	1.081	0.954
A2	0.954	0.539	1.081	0.539	0.539
A3	0.44	0.287	0.44	0.287	0.111
A4	1.333	0.954	1.174	0.954	0.954
A5	0.954	0.954	1.488	0.539	0.539
A6	0.174	0.22	0.27	0.141	0.287
A7	0.44	0.111	0.27	0.111	0.111
A8	0.44	0.111	0.347	0.111	0.111
A9	1.081	0.287	0.22	0.539	0.174
A10	1.333	0.22	0.44	0.44	0.440
A11	0.174	0.44	0.347	0.539	0.347
A12	0.174	0.44	0.22	0.27	0.270
A13	0.347	0.111	0.27	0.22	0.270
A14	0.347	0.22	0.44	0.141	0.287
A15	0.44	0.347	0.22	0.287	0.111
Geometry variables (in.)					
X2	123.189	133.612	118.346	101.5775	128.4213
X3	231.595	234.752	225.209	227.9112	246.3209
Y2	107.189	100.449	119.046	134.7986	123.4423
Y3	119.175	104.738	105.086	128.2206	116.0383
Y4	60.462	73.762	63.375	54.863	51.7145
Y6	–16.728	–10.067	–20	–16.4484	–11.242
Y7	15.565	-1.339	-20	-13.3007	-17.662
Y8	36.645	50.402	57.722	54.8572	50.5825
Weight (lb)	120.528	79.82	104.573	76.6854	74.692

Table 2. Bounds of geometry variables of example 1

Design variable (in.)	**Lower bound**	**Upper bound**
X2	100	140
X3	220	260

Y2	100	140
Y3	100	140
Y4	50	90
Y6	–20	20
Y7	–20	20
Y8	20	60

5.3. Example 2: Eighteen-bar truss structure

The 18-bar truss structure, shown in Figure 2, is chosen for both sizing and geometry optimization. Five vertical loads of 20 kips (88.964 kN) are acting on nodes 1, 2, 4, 6 and 8. The material density is 0.1 lb/in.^{3} (2767.99 kg/m^3) and the modulus of elasticity, E, is 10,000 ksi (68,947.6 MPa). The stress limit is 20 ksi(137.895 MPa) in both tension and compression for all members. The Euler buckling strength for the *i*-th member with a cross-sectional area of A_i and length of L_i is determined by $-4EA_i / L_i^2$, (i = 1, 2, . . . , 18). The members of the structure are linked into 4 groups, considered as 4 sizing variables. The cross-sectional areas of members are chosen from the set: S = {2, 2.25, 2.5, . . . , 21.25, 21.5, 21.75} in.^{2}. Nodes 3, 5, 7 and 9 are allowed to move in both x and y directions. In this case 8 geometry variables are added to the problem. Therefore there are 12 design variables in this example.

The boundaries of geometry variables are given in Table 3. The results of optimization are given in Table 4.

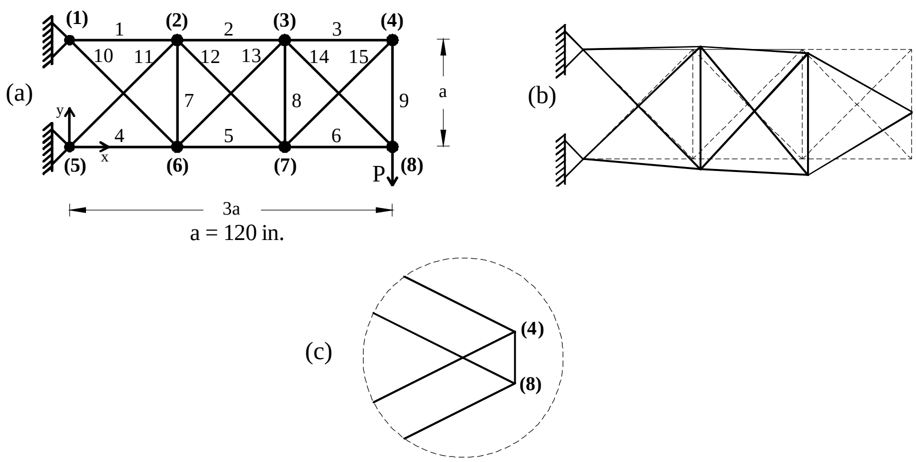


Figure 1. (a) Fifteen-bar truss structure; (b) Optimum layout of the 15-bar truss; (c) Position of the nodes, 4 and 8

Table 3. Bounds of geometry variables of example 2

Design variable (in.)	Lower bound	Upper bound
X3	775	1225

X5	525	975
X7	275	725
X9	25	475
Y3	–225	245
Y5	–225	245
Y7	–225	245
Y9	–225	245

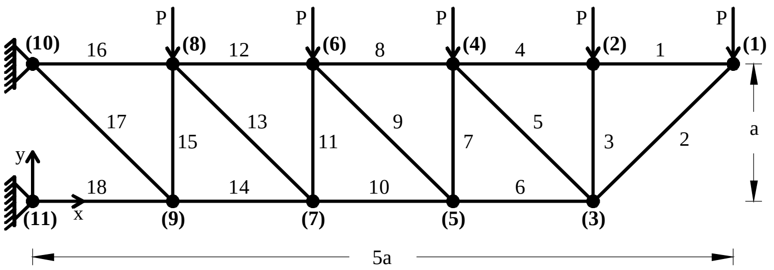


Figure 2a. Eighteen-bar truss structure, a = 250 in

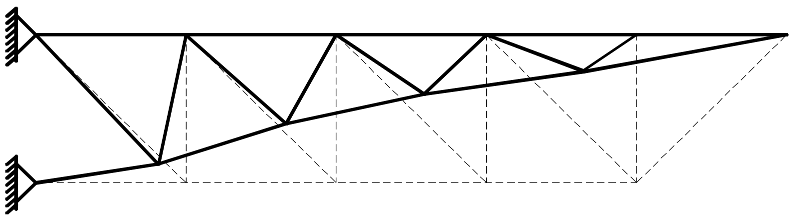


Figure 2b. Optimum layout of the 18-bar truss

5.4. Example 3: Twenty five-bar space truss

The sizing and geometry optimization of the 25-bar space truss (Figure 3) in considered in this example. The loading data is given in Table 5. The stress limit is 40 ksi (275.79 MPa) in both tension and compression for all members, and the displacement of all nodes in directions x, y and z is limited to ±0.35 in. (±0.889 cm). The density of the material is 0.1 lb/in.^{3} (2767.99 kg/m^3) and the modulus of elasticity is 10,000 ksi (68,947.6 MPa). As shown in Table 7, the members of the truss are linked into 8 groups, considered as 8 sizing variables. The sizing variables are chosen from the following set: S = {0.1a (a = 1, . . . , 26), 2.8, 3, 3.2, 3.4} in.^{2}. For geometry optimization, the nodes 3, 4, 5 and 6 are allowed to move in all x, y and z directions, and the nodes 7, 8, 9 and 10 are allowed to move only in x and y directions. Since the structure is symmetric, there are 5 geometry variables ($x_4 = x_5 = -x_3 = -x_6$, $x_8 = x_9 = -x_7 = -x_{10}$, $y_3 = y_4 = -y_5 = -y_6$, $y_7 = y_8 = -y_9 = -y_{10}$, $z_3 = z_4 = z_5 = z_6$) in this example. The limits of geometry variables and the results of optimization are given in Tables 6 and 7 respectively.

Table 4. Comparison of results for the eighteen-bar truss structure

Design variables	Members	Hasançebi and Erbatur [19]	Kaveh and Kalatjari [20]	Rahami et al. [18]	The present work
Sizing variables (in.^{2})					
G1	1, 4, 8, 12, 16	12.50	12.25	12.75	12.5
G2	2, 6, 10, 14, 18	18.25	18	18.5	18
G3	3, 7, 11, 15	5.5	5.25	4.75	5.25
G4	5, 9, 13, 17	3.75	4.25	3.25	3.75
Geometry variables (in.)					
X3		933	913	917.4475	913.6544
Y3		188	186.8	193.7899	188.0802
X5		658	650	654.3243	646.7496
Y5		148	150.5	159.9436	149.8965
X7		422	418.8	424.4821	416.7127
Y7		100	97.4	108.5779	99.8661
X9		205	204.8	208.4691	204.1377
Y9		32	26.7	37.6349	31.5643
Weight (lb)		4574.28	4547.9	4530.7	4527.96

Table 5. Loading of spatial 25-bar truss

Node	Fx (kips)	Fy (kips)	Fz (kips)
1	1	–10	–10
2	0	–10	–10
3	0.5	0	0
6	0.6	0	0

Table 6. Bounds of geometry variables of example 3

Design variable (in.)	Lower bound	Upper bound
X4	20	60
Y4	40	80
Z4	90	130
X8	40	80
Y8	100	140

Table 7. Comparison of results for the spatial 25-bar truss structure

Design variables	Members	Wu and Chow [15]	Tang et al. [16]	Kaveh and Kalatjari [20]	Rahami et al. [18]	The present work

Sizing variables (in. ^{2})						
G1	1	0.1	0.1	0.1	0.1	0.1
G2	2, 3, 4, 5	0.2	0.1	0.1	0.1	0.1
G3	6, 7, 8, 9	1.1	1.1	1.1	1.1	1
G4	10, 11	0.2	0.1	0.1	0.1	0.1
G5	12, 13	0.3	0.1	0.1	0.1	0.1
G6	14, 15, 16, 17	0.1	0.2	0.1	0.1	0.1
G7	18, 19, 20, 21	0.2	0.2	0.1	0.2	0.1
G8	22, 23, 24, 25	0.9	0.7	1	0.8	0.9
Geometry variables (in.)						
X4		41.07	35.47	36.23	33.0487	37.5729
Y4		53.47	60.37	58.56	53.5663	54.4903
Z4		124.6	129.07	115.59	129.9092	130
X8		50.8	45.06	46.46	43.7826	51.8904
Y8		131.48	137.04	127.95	136.8381	139.5662
Weight (lb)		136.2	124.94	124	120.1149	117.264

5.5. Example 4: One hundred twenty-bar dome truss

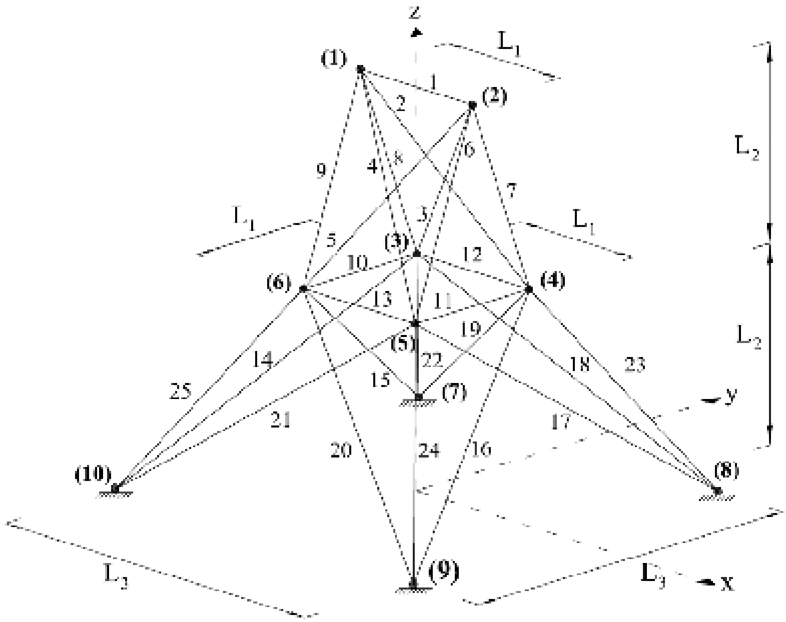
The sizing and geometry optimization of the 120-bar dome truss, shown in Figure 4, is performed in [21]. Here, only the sizing optimization of the structure is considered. The structure is subjected to vertical loading at all unsupported nodes. The loads are taken as -13.49 kips (-60 kN) at node 1, -6.744 kips (-30 kN) at nodes 2 to 14, and -2.248 kips (-10 kN) in the rest of the nodes. The minimum allowable cross-sectional area of each member is limited to 0.775 in.^{2} (5 cm^2). The allowable tensile stress is 0.6F_y and the compressive stress constraint σ_i^b of member *i* is as follows [22]:

$$\sigma_i^b = \begin{cases} \left[\left(1-\dfrac{\lambda_i^2}{2C_c}\right)F_y\right] \Big/ \left(\dfrac{5}{3}+\dfrac{3\lambda_i}{8C_c}-\dfrac{\lambda_i^3}{8C_c^3}\right) & \text{for } \lambda_i < C_c \\ \dfrac{12\pi^2 E}{23\lambda_i^2} & \text{for } \lambda_i \geq C_c \end{cases} \tag{11}$$

where F_y is the yield stress of steel, E is the modulus of elasticity, λ_i is the slenderness ratio ($\lambda_i = kL_i/r_i$), k is the effective length factor, L_i is the length of the member, r_i is the radius of gyration, and $C = \sqrt{2\pi^2 E/F_y}$. Here, the material density is 0.288 lb/in.^{3} (7971.81 kg/m^3), F_y = 58 ksi (400 MPa), E = 30,450 ksi (210,000 MPa), and $r_i = 0.4993A_i^{0.6777}$ for the pipe sections [6]. In this example, two cases of displacement constraints are considered.

Case 1: no displacement constraints is imposed;

Case 2: the displacement of all nodes in directions x, y and z is limited to ±0.1969 in. Table 8 gives the results of optimization for both cases.

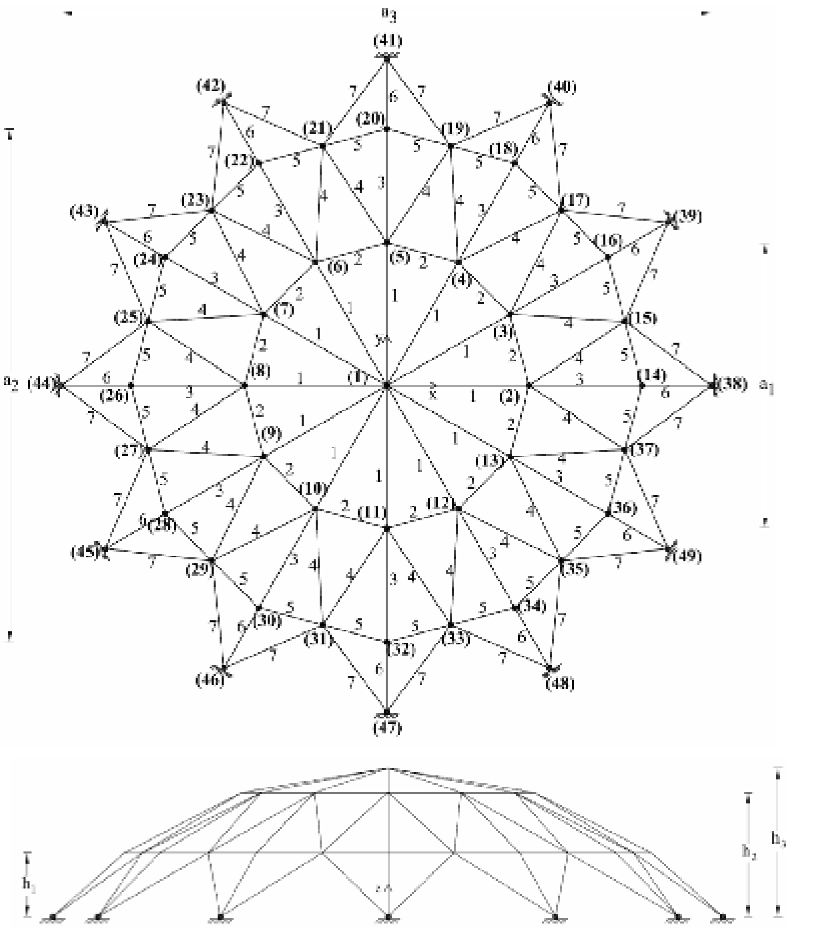


L_1 = 75 in. L_2 = 100 in. L_3 = 200 in.

Figure 3. Spatial twenty five-bar truss

Table 8. Comparison of results for the 120-bar dome truss

Case 1			Case 2	
Design variables	**Lee and Geem [6]**	**The present work**	**Lee and Geem [6]**	**The present work**
	Sizing variables (in.^{2})			
A1	3.295	3.3293	3.296	3.3005
A2	2.396	2.4384	2.789	2.7481
A3	3.874	4.0168	3.872	3.9036
A4	2.571	2.5918	2.57	2.5713
A5	1.15	1.1823	1.149	1.2889
A6	3.331	3.4513	3.331	3.4089
A7	2.784	2.7854	2.781	2.8150
Weight (lb)	19707.77	20016.67	19893.34	20125.35



a_1 = 546.61 in. a_2 = 984.252 in. a_3 = 1251.02 in.

h_1 = 118.11 in. h_2 = 230.315 in. h_3 = 275.591 in.

Figure 4. One hundred twenty-bar dome truss

6. CONCLUSION

Firefly algorithm is a novel nature-inspired algorithm based on the flashing characteristics of fireflies. In the present study, an optimization method based on the firefly algorithm is proposed. Both sizing and geometry optimization of different types of truss structures under stress, displacement and buckling constraints are carried out and numerical results are compared to the previously reported results in the literature. Additionally, for all examples, the general performance of the algorithm in 50 runs is reported (see Table 9). Numerical results indicate the robustness and efficiency of the proposed method in optimum design of truss structures. However, further research is required in order to determine the performance of firefly algorithm based methods in topology optimization of truss structures.

Table 9. General performance of the algorithm in 50 runs

Example	Number of structural analyses	Minimum weight (lb)	Mean weight (lb)	Maximum weight (lb)	Standard deviation (lb)
15-bar truss	10000	74.692	81.0246	87.4441	3.01
18-bar truss	10000	4527.96	4575.19	4642.9	31.36
25-bar truss	10000	117.264	118.98	125.23	2.26
120-bar dome (Case1)	15000	20016.67	20197.68	20374.4	97.14
120-bar dome (Case2)	15000	20125.35	20297.51	20484.37	104.09

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