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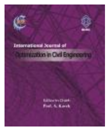

OPTIMAL DESIGN OF DOUBLE LAYER GRIDS CONSIDERING NONLINEAR BEHAVIOUR BY SEQUENTIAL GREY WOLF ALGORITHM

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ABSTRACT

The present paper tackles the optimization problem of double layer grids considering nonlinear behaviour. In this paper, an efficient optimization algorithm is proposed to achieve the optimization task based on the newly developed grey wolf algorithm (GWA) termed as sequential GWA (SGWA). In the framework of SGWA, a sequence of optimization processes is implemented in which the initial population of each process is selected from the neighboring region of the best design found in the previous optimization process. This procedure is repeated until a termination criterion is met. Two illustrative examples are presented and optimization is performed by GWA and SGWA and two other meta-heuristics. The numerical results indicate that the proposed SGWA utperforms the other algorithms in finding optimal design of nonlinear double layer grids.

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KEY WORDS: design optimization; nonlinear behaviour; double layer grid; meta-heuristic.

1. INTRODUCTION

As the space structures are employed to cover wide span column free areas, they have a huge number of structural elements and therefore, sufficient attention must be paid to systematic designing of these structures. For this purpose, design of space structures can be conveniently achieved by employing optimization techniques. It is obvious that an optimal design has a great influence on the economy and safety of all types of the structures. In this case, optimizing space structures results in more efficient structural configurations [1]. The

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present paper is devoted to design optimization of double layer grids as a simple and popular system of space structures considering nonlinear behaviour. Such optimization problem is computationally complex and it is necessary to use an efficient optimization algorithm for dealing with this problem. Many of gradient-based optimization algorithms have difficulties when dealing with such type of problems, and they may converge to local optima. To overcome these difficulties, utilizing algorithms possessing global search ability is inevitable. In contrast with gradient-based optimization algorithms, meta-heuristics can be efficiently employed to tackle complex optimization problems. Meta-heuristics are designated on the basis of stochastic natural phenomena, and they have attracted a great deal of attention during the last two decades. As the meta-heuristic optimization techniques require no gradient computations, they are simple for computer implementation [2]. In the recent years, many meta-heuristic algorithms have been designed by researchers and many successful applications of them have been reported in the field of structural optimization. One of the newly developed meta-heuristics is grey wolf algorithm (GWA) [3] which is designed based on the simulation of leadership hierarchy and hunting mechanism of grey wolves in nature. The work of Mirjalili *et. al*. [3] demonstrated the superiority of GWA over the particle swarm optimization (PSO), gravitational search algorithm (GSA), differential evolution (DE), evolutionary programming (EP), and evolution strategy (ES).

Optimization of space structures based on linear behavior has been achieved in several researches [4-8] during the last years. Saka and Ulker [9] and Saka and Kameshki [10] optimized space structures considering only geometrical nonlinearity. They employed gradient-based methods for solving the optimization problem. In the recent work of Kamyab and Salajegheh [1] material and geometrical nonlinearities were considered in optimization of scallop domes subject to static loading. They employed an enhanced particle swarm optimization algorithm for performing optimization process.

In the present study, an efficient version of GWA, called here as sequential grey wolf algorithm (SGWA), is proposed to implement the complex optimization problem of double layer grids considering nonlinear behaviour. Two design examples are presented and the numerical results demonstrate the efficiency of the proposed SGWA in comparison with the GWA, harmony search algorithm (HSA) [11] and firefly algorithm (FA) [12].

2. NONLINEAR BEHAVIOR

The accurate and computationally efficient model for members of double layer grids is required for simulation of realistic behaviour of the structures. For this purpose, a 3-D uniaxial co-rotational truss element exists in the OpenSees [13] platform is utilized to model structural elements. This finite element has plasticity and large deflection capabilities. In elasto-plastic analysis the Von Mises yield function is used as yield criterion. Flow rule in this model is associative and the hardening rule is Bi-linear kinematics hardening in tension. In compression, according to FEMA274 [14], it is assumed that the element buckles at its corresponding buckling stress state and its residual stress is about 20% of the buckling stress. In this case, the stress-strain relation is shown in Fig. 1. In this figure, σ_b, and σ_y are buckling, and yield stresses, respectively and ε_b, and ε_y are their corresponding strains.

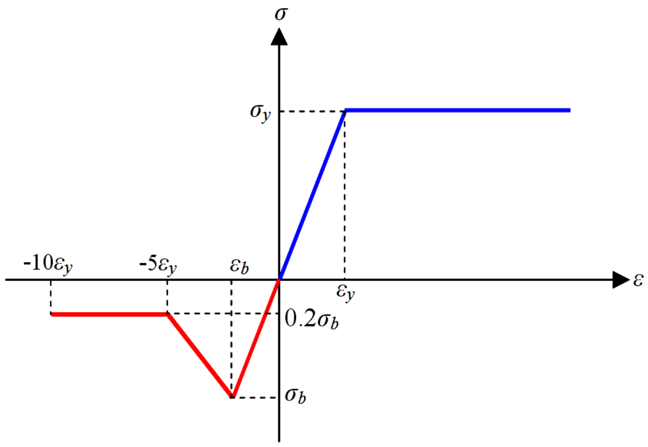

Figure 1. Stress-strain behaviour of 3-D uniaxial co-rotational truss element

In this model, buckling stress of the elements is computed as follows [15]:

$$\sigma_b = \begin{cases} (0.658^{\lambda_c^2})\sigma_y & \lambda_c \le 1.5 \\ (\dfrac{0.877}{\lambda_c^2})\sigma_y & \lambda_c > 1.5 \end{cases} \quad , \quad \lambda_c = \frac{KL}{r\pi}\sqrt{\frac{\sigma_y}{E}} \tag{1}$$

where λ_c is slenderness parameter; E is modulus of elasticity; and K is effective length factor which for space structure elements is chosen to be 1.

3. FORMULATION OF PROBLEM

The main aim of the sizing optimization problem of double layer grids considering nonlinear behavior is to minimize the weight of the structure, subject to two constraints. The first constraint limits the maximum deflection of the structure while the second one is to ensure the overall stability of the structure during the optimization process.

For a double layer grid with *ne* members collected in *ng* groups, if the design variables associated with each design group are selected from a given profile list, the nonlinear optimization problem can be formulated as follows:

$$\text{Minimize: } w(X) = \sum_{i=1}^{ne} \rho_i A_i \sum_{j=1}^{nm} L_j \tag{2}$$

$$\text{Subject to: } g_d(X) = \frac{d_{\text{max}}}{d_{\text{all}}} - 1 \le 0 \tag{3}$$

$$g_S(X) = \frac{f_{\text{app}}}{f_u} - 1 \le 0 \tag{4}$$

$$X = \{X_1 \quad X_2 \quad \dots \quad X_i \quad \dots \quad X_{ng}\}^{\mathrm{T}} \tag{5}$$

where w represents the weight of the frame, ρ_i and A_i are weight of unit volume and cross-sectional area of the ith group section, respectively, nm is the number of elements collected in the ith group, L_j is the length of the jth element in the ith group, d_{max} is the maximum deflection of the structure and d_{all} is its allowable value, f_{app} is applied load and f_u is ultimate load of the structure which can be determined by incremental nonlinear analysis, X_i is an integer value expressing the sequence numbers of steel sections assigned to the ith group.

In this study, the constraints of the optimization problem are handled using the concept of exterior penalty functions method (EPFM) [16]. In this case, the pseudo unconstrained objective function is expressed as follows:

$$\Phi(X,r) = w(X)\left(1 + r\left(\max\{0,\, g_d(X)\}\right)^2 + r\left(\max\{0,\, g_S(X)\}\right)^2\right) \quad (6)$$

where Φ and r are the pseudo objective function and penalty parameter, respectively.

In this study, the above pseudo objective function is minimized by HSA, FA, GWA and SGWA meta-heuristics and the results are compared.

4. META-HEURISTICS

Meta-heuristics are popular optimization tools in various research areas due to their simplicity, flexibility and ease of computer implementation. One of the recent addition to the meta-heuristics is GWA. The computational merits of GWA with respect to genetic algorithm (GA) and HSA have been demonstrated for tackling structural optimization problems in [17]. In the present study, a sequential GWA (SGWA) is propsed to tackle the optimiztion problem of double layer grids considering nonlinear behaviour and its results are compared with those of HSA, FA and GWA. As regrds HSA and FA are well-known meta-heuristics, their theoretical backgrounds are not explained here and only GWA and SGWA are explained in the next sections.

4.1 Grey wolf algorithm

GWA is a new metaheuristic and has been proposed by Mirjalili *et al*. [3] based on the leadership hierarchy and hunting mechanism of grey wolves in nature. In the process of GWA, the leadership hierarchy and the hunting process are simulated.

In the wolf's pack the leaders are usually a male and a female, called alpha and their decisions are dictated to the pack. The second level in the hierarchy of grey wolves is beta. The betas are subordinate wolves that help the alpha in decision-making or other pack activities. The beta wolf is probably the best candidate to be the alpha in case one of the alpha wolves passes away or becomes very old. The lowest ranking grey wolf is omega. The omega plays the role of scapegoat. Omega wolves always have to submit to all the other dominant wolves. They are the last wolves that are allowed to eat. If a wolf is not an alpha, beta, or omega, he/she is called subordinate (or delta in some references). Delta wolves have to submit to alphas and betas, but they dominate the omega [3].

Hunting as another social behavior of grey wolfs comprises three main phases. The first phase includes tracking, chasing, and approaching the prey. In the second one the grey wolfs pursue, encircle, and harass the prey until it stops moving and in the last phase they attack towards the prey [3]. In [3] the mentioned hunting technique and the social hierarchy of grey wolves have been mathematically modeled to propose GWA.

In designing GWA, the first, second and third best solutions are considered as α, β and δ wolves, respectively while the rest of the candidate solutions are considered as ω. In the framework of GWA, ω wolves follow α, β and δ wolves during the optimization process.

The following equations are used to model the encircling behavior of grey wolves [3]:

$$A_i = 2\bar{A}_i . R_{1i} - \bar{A}_i \tag{7}$$

$$C_i = 2R_{2i} \tag{8}$$

$$D_i = \left| C_i . X_p^t - X_i^t \right| \tag{9}$$

$$X_i^{t+1} = X_p^t - A_i . D_i \tag{10}$$

where R_{1i} and R_{2i} are random vectors in [0,1]; $\bar{A}_i$ is a vector that its components are linearly decreased from 2 to 0 during the optimization process; A_i and C_i are coefficient vectors; X_p^t is the prey in iteration t; X_i^t is the ith grey wolf in iteration t.

For simulation of the hunting behavior of grey wolves, it is supposed that the alpha, beta, and delta have better knowledge about the potential location of prey. Therefore, the first three best solutions obtained so far should be saved and the other wolves in the pack update their positions according to the position of the best ones (around the prey) as follows [3]:

$$D_\alpha = \left| C_1 . X_\alpha^t - X^t \right| \tag{11}$$

$$D_\beta = \left| C_2 . X_\beta^t - X^t \right| \tag{12}$$

$$D_\delta = \left| C_3 . X_\delta^t - X^t \right| \tag{13}$$

$$X_1^t = X_\alpha^t - A_1 . D_\alpha \tag{14}$$

$$X_2^t = X_\beta^t - A_2 . D_\beta \tag{15}$$

$$X_3^t = X_\delta^t - A_3 . D_\delta \tag{16}$$

$$X^{t+1} = \frac{1}{3} \sum_{j=1}^{3} X_j^t \tag{17}$$

The final step in hunting process of grey wolfs is attacking prey as soon as it stops moving. In GWA, decreasing the values of $\bar{A}_i$ components from 2 to 0 during the optimization process simulates approaching the prey and provides the exploration ability of the algorithm. Also, the exploitation ability of the GWA comes from the random components of the C vector.

4.2 Sequential Grey wolf algorithm

The important aspects of all meta-heuristics are exploration and exploitation and balancing these abilities can play a major role in convergence behaiour of the algorithms. In the standard version of GWA the exploration and exploitation abilities are not balanced and the algorithm may converge to local optima in complex optimization problems. In order to improve the convergence rate of the GWA a modification is accomplished in this paper. In fact, the proposed modification is a sequential implementation of GWA-based optimization processes. The proposed algorithm in the present study is termed as sequential GWA (SGWA). In the first stage of SGWA, *nw* search agents are randomly selected from search space and the GWA is employed to achieve an optimization process. In this process the best solution is saved as X_{best}. In the next step, a new wolf's pack is selected from the neighboring region of the found X_{best}. In this case, X_{best} is transformed to the new pack and the remaining ones are randomly selected as follows:

$$X_j = \boldsymbol{N}(X_{\text{best}}, \xi X_{\text{best}}), \; j = 1,2,...,(nw-1) \tag{18}$$

where $\boldsymbol{N}(X_{\text{best}}, \xi X_{\text{best}})$ represents a random normally distributed vector with the mean X_{best} and the standard deviation ξX_{best}. The parameter ξ plays an important role in convergence behavior of the algorithm and based on the computational experiences of the previous works [2, 18] the best value for this parameter is equal to 0.1.

The produced pack is employed by GWA to achieve another optimization process and this procedure is repeated for *nt* times and the best solution found in this manner is the final solution of the algorithm. In this paper, *nt* is chosen to be 4.

5. NUMERICAL EXAMPLES

In the present work, two double layer grids with 200 and 968 bars are considered as the illustrative examples. For both the structures, Young's modulus, mass density and yield stress are 2.1×10^{10} kg/m^2, 7850 kg/m^3 and 3.515×10^7 kg/m^2, respectively. A uniformly distributed load of 250 kg/m^2 is applied on the top layer of the structures.The design variables are selected from a set of standard Pipe profiles listed in Table 1.

Table 1: The available list of standard Pipe profiles

NO.	Profile	*A* (cm^2)	*r* (*cm*)	NO.	Profile	*A* (cm^2)	*r* (*cm*)
1	D33.70x2.6	2.540	1.1000	**14**	D159.0x4.0	19.480	5.4814
2	D48.30x2.6	3.730	1.6200	**15**	D168.3x4.0	20.65	5.8102
3	D60.30x3.2	5.740	2.0200	**16**	D193.7x4.5	26.75	6.6922
4	D76.10x3.2	7.329	2.5799	**17**	D219.1x5.0	33.63	7.5716
5	D82.50x3.2	7.972	2.8060	**18**	D244.5x5.4	40.56	8.4557
6	D88.90x3.2	8.616	3.0321	**19**	D273.0x5.6	47.04	9.4570
7	D101.6x3.6	11.080	3.4672	**20**	D298.5x5.9	54.23	10.3471
8	D108.0x3.6	11.810	3.6934	**21**	D323.9x5.9	58.94	11.2450
9	D114.3x3.6	12.520	3.9161	**22**	D355.6x6.3	69.13	12.3536
10	D127.0x4.0	15.450	4.3504	**23**	D368.0x6.3	71.59	12.7895
11	D133.0x4.0	16.210	4.5629	**24**	D406.4x6.3	79.19	14.1475
12	D139.7x4.0	17.050	4.8004	**25**	D419.0x7.1	91.88	14.5645
13	D152.4x4.0	18.650	5.2483	**26**	D457.2x7.1	100.4	15.9150

5.1 Example 1: The 200-bar double layer grid

The configuration of the 200-bar double layer grid, supported at corner nodes, is shown in Fig. 2. Grouping details of the structural members are depicted in Fig. 3.

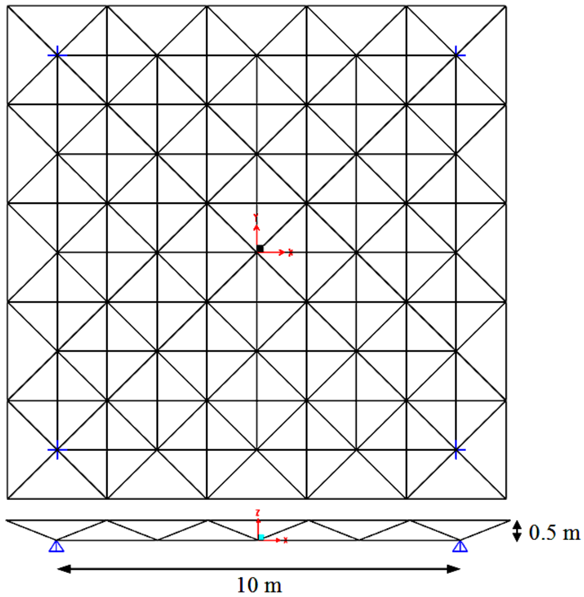

Figure 2. Configuration of the 200-bar double layer grid

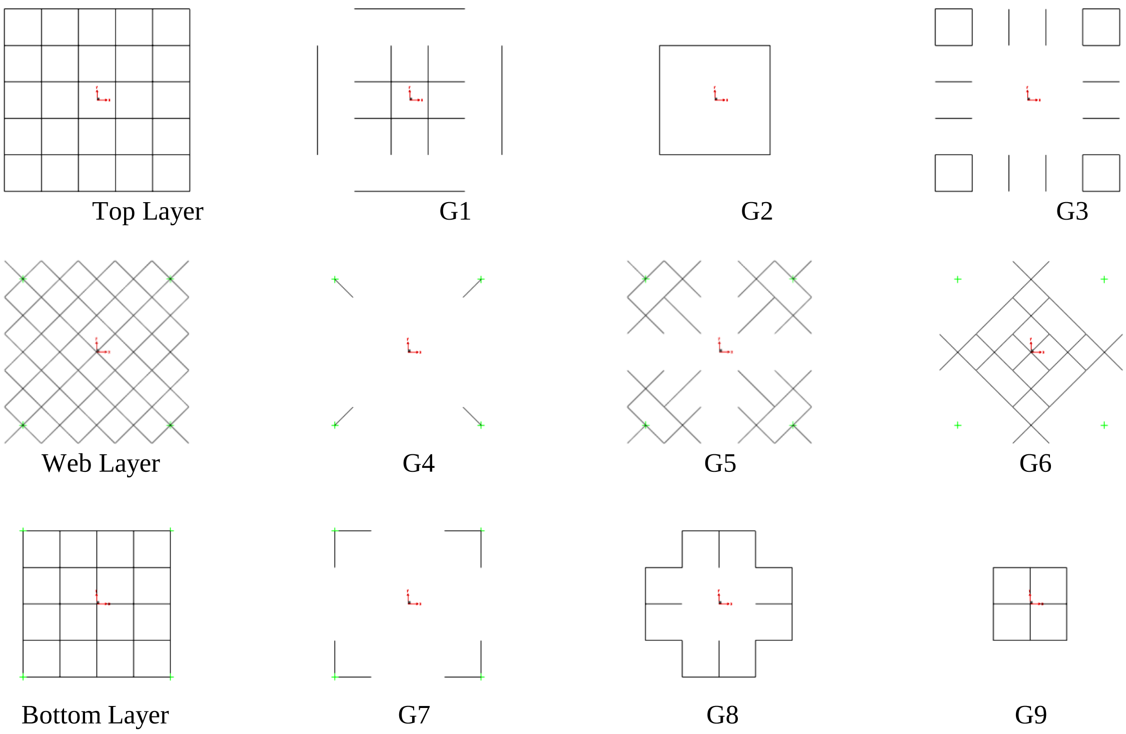

Figure 3. Member grouping details of the 200-bar double layer grid

For performing optimization process, the number of search agants in the pack is chosen to be 20 and the maximum number of iterations is limited to 200. For SGWA, 4 optimization proceses with 50 iterations are implemented. In this example, d_{all}=7.5 cm.

The results of optimization considering nonlinear behaviour are compared in Table 2 with those of Engineering Design found by SAP2000 [20] based on AISC-LRFD [15] considering linear behaviour. In this table, maximum demand-capacity ratio (*DCR*) of structural members is represented by DCR_{max}.

Table 2: Engineering and optimum designs of the 200-bar double layer grid

Design variables	Engineering Design	Optimal Designs			
		HS	**FA**	**GWA**	**SGWA**
G1	D76.10x3.2	D76.10x3.2	D60.30x3.2	D60.30x3.2	D60.30x3.2
G2	D101.6x3.6	D101.6x3.6	D108.0x3.6	D108.0x3.6	D108.0x3.6
G3	D33.70x2.6	D33.70x2.6	D33.70x2.6	D33.70x2.6	D33.70x2.6
G4	D101.6x3.6	D101.6x3.6	D114.3x3.6	D101.6x3.6	D101.6x3.6
G5	D60.30x3.2	D60.30x3.2	D60.30x3.2	D60.30x3.2	D60.30x3.2
G6	D33.70x2.6	D33.70x2.6	D33.70x2.6	D33.70x2.6	D33.70x2.6
G7	D76.10x3.2	D33.70x2.6	D33.70x2.6	D48.30x2.6	D33.70x2.6
G8	D48.30x2.6	D48.30x2.6	D48.30x2.6	D48.30x2.6	D48.30x2.6
G9	D33.70x2.6	D33.70x2.6	D33.70x2.6	D33.70x2.6	D33.70x2.6
Weight (kg)	1702.73	1627.57	1578.25	1588.66	1569.94
f_u (kg/m^2)	287.5	280.0	276.2	277.5	275.0
d_{max} (cm)	6.57	7.50	7.50	7.50	7.50
DCR_{max}	0.999	-	-	-	-

The numerical results reveal that among the employed meta-heuristics, SGWA finds the best solution which is 3.54%, 0.53%, and 1.18% lighter than those of found by HA, FA, and GWA, respectively. Furthermore, the weight of the optimum design found by SGWA is 7.80% lighter than the weight of the engineering design.

It should be noted that, in the engineering design DCR_{max} dominates the design while in the nonlinear optimal designs d_{max} is the active constraint. Moreover, it can be observed that however optimal designs are lighter than the engineering design, their ultimate load (f_u) is very close to that of the engineering design.

5.2 Example 2: The 968-bar double layer grid

Fig. 4. Shows the configuration of the 200-bar double layer grid. The span of the structure and its height are 30 m and 1.5 m, respectively. As shown in this figure the structure is supported at four corner nodes of the bottom layer. Figs. 5, 6, and 7 present the grouping details of the structural members in top, web and bottom layers, respectively.

In the present example, the engineering design is achieved by SAP2000 software. The pack size, the maximum number of iterations and the number of optimization process for SGWA meta-heuristic are same as the first example. In this example, the allowable deflection is set to 27.5 cm.

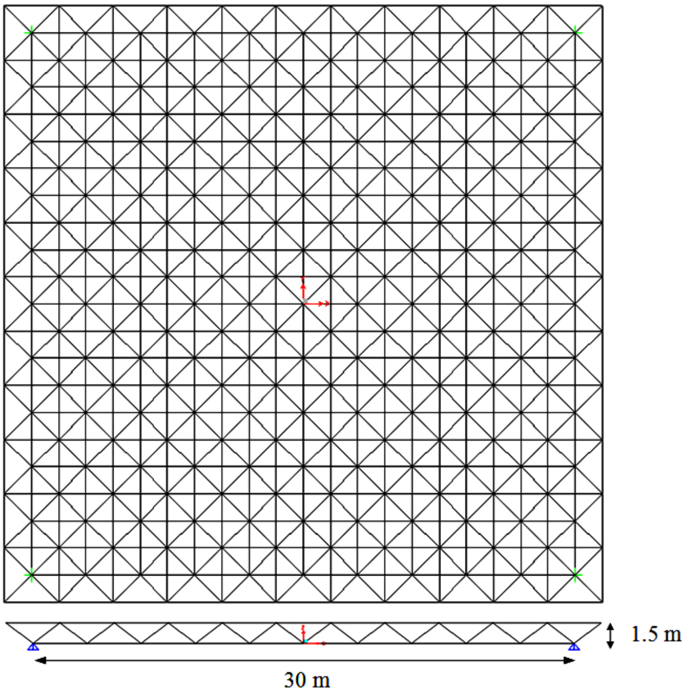

Figure 4. Configuration of the 968-bar double layer grid

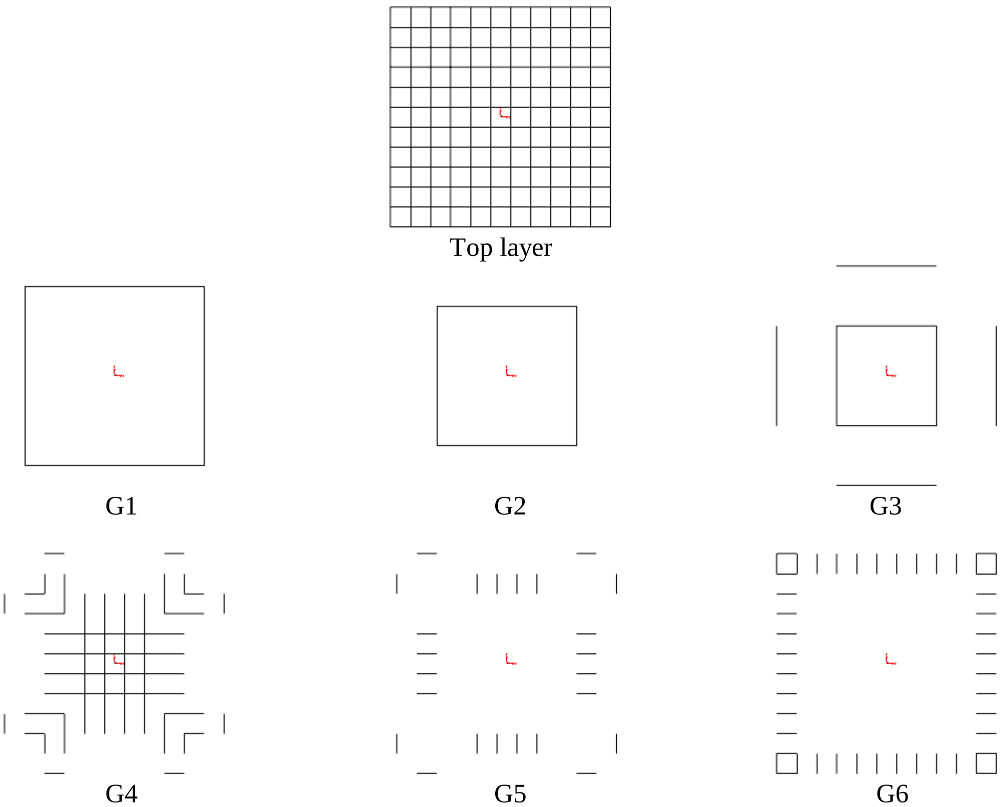

Figure 5. Member grouping details of the top layer for the 968-bar double layer grid

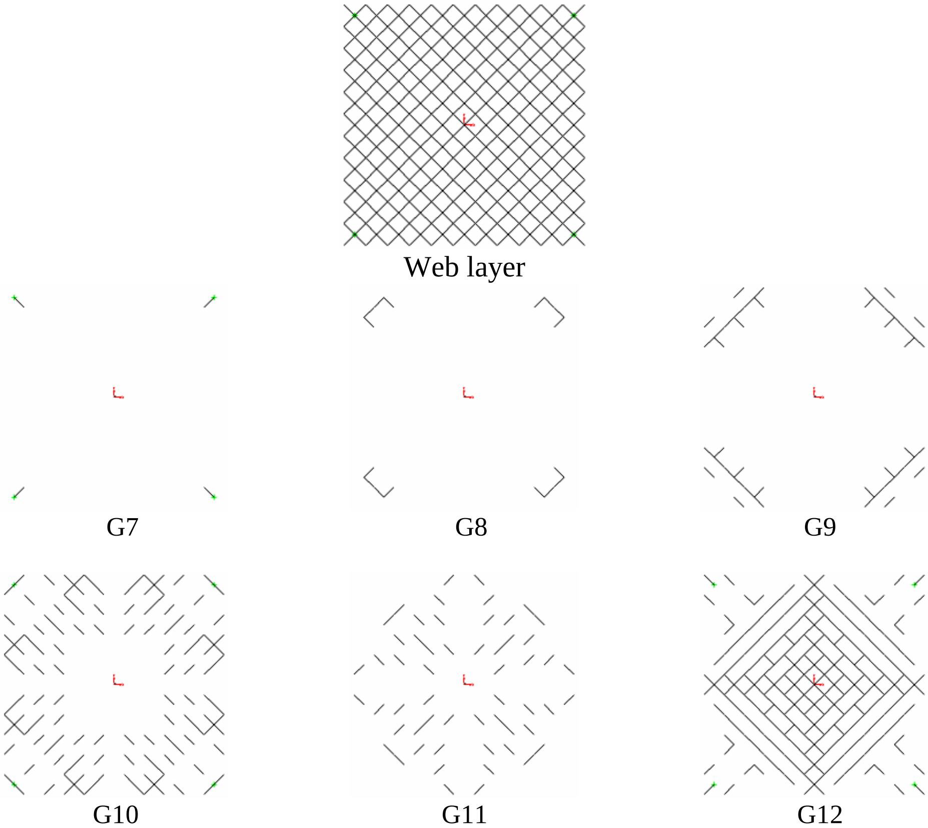

Figure 6. Member grouping details of the web layer for the 968-bar double layer grid

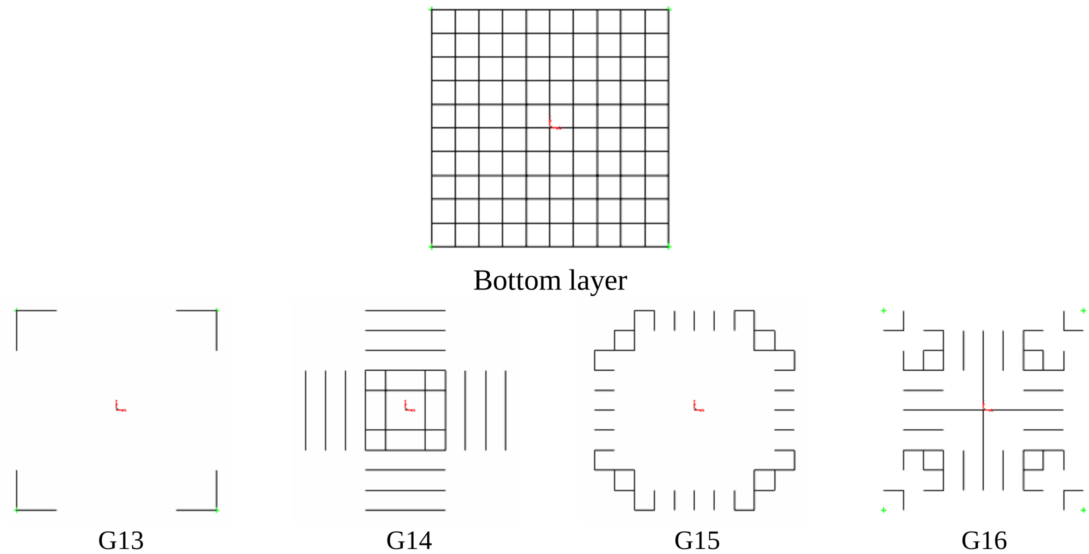

Figure 7. Member grouping details of the bottom layer for the 968-bar double layer grid

Table 3 compares the optimal solutions found by various meta-heuristics with that of Engineering Design.

Table 3: Engineering and optimum designs of the 968-bar double layer grid

Design variables	Engineering Design	Optimal Designs			
		HS	**FA**	**GWA**	**SGWA**
G1	D244.5x5.4	D273.0x5.6	D273.0x5.6	D273.0x5.6	D273.0x5.6
G2	D193.7x4.5	D168.3x4.0	D152.4x4.0	D168.3x4.0	D193.7x4.5
G3	D193.7x4.5	D168.3x4.0	D193.7x4.5	D168.3x4.0	D159.0x4.0
G4	D127.0x4.0	D114.3x3.6	D114.3x3.6	D114.3x3.6	D108.0x3.6
G5	D82.50x3.2	D82.50x3.2	D76.10x3.2	D82.50x3.2	D76.10x3.2
G6	D33.70x2.6	D48.30x2.6	D33.70x2.6	D33.70x2.6	D33.70x2.6
G7	D244.5x5.4	D406.4x6.3	D244.5x5.4	D406.4x6.3	D244.5x5.4
G8	D139.7x4.0	D168.3x4.0	D152.4x4.0	D168.3x4.0	D139.7x4.0
G9	D114.3x3.6	D108.0x3.6	D101.6x3.6	D108.0x3.6	D101.6x3.6
G10	D88.90x3.2	D88.90x3.2	D82.50x3.2	D88.90x3.2	D76.10x3.2
G11	D60.30x3.2	D60.30x3.2	D60.30x3.2	D60.30x3.2	D48.30x2.6
G12	D60.30x3.2	D48.30x2.6	D48.30x2.6	D48.30x2.6	D48.30x2.6
G13	D273.0x5.6	D273.0x5.6	D244.5x5.4	D273.0x5.6	D244.5x5.4
G14	D101.6x3.6	D108.0x3.6	D101.6x3.6	D108.0x3.6	D114.3x3.6
G15	D76.10x3.2	D60.30x3.2	D60.30x3.2	D60.30x3.2	D60.30x3.2
G16	D82.50x3.2	D76.10x3.2	D82.50x3.2	D76.10x3.2	D88.90x3.2
Weight (kg)	25426.28	23746.62	23136.76	23611.82	22775.29
f_u (kg/m^2)	282.5	278.5	277.5	278.5	275.0
$d_{\max}$ (cm)	26.57	27.50	27.50	27.50	27.50
$DCR_{\max}$	0.999	-	-	-	-

It can be easily observed that the computational performance of the proposed SGWA is better than that of the other employed meta-heuristics. The structural weight of optimal design found by SGWA is 4.09%, 1.56%, and 3.54% lighter than the optimal weights found by HA, FA, and GWA, respectively. In addition, the SGWA converges to an optimal design which is 10.43% lighter than that of the engineering design.

The results of Table 3 indicate that the active design constraints for engineering and optimal designs are $DCR_{\max}$ and $d_{\max}$, respectively. Finally, the values of f_u computed for optimal designs and engineering design imply that however the weight of the optimal designs are less than that of the engineering design their ultimate loads are almost the same.

6. CONCLUSIONS

Optimal design of double layer grids considering nonlinear behavior is the main aim of the present study. In order to simulate the realistic behaviour of the structures, the structural members are modelled by a finite element with plasticity and large deflection capabilities. The discrete design variables are selected from a list of standard sections. As regards that the standard version of meta-heuristics are not usually efficient for solving complex optimization problems, an efficient optimization algorithm based on sequential implementation of GWA is proposed to deal with the optimization problem. The proposed algorithm, named as SGWA, is a multi-stage implementation of GWA in which the initial pack of each stage is generated based on the best solution found in the previous stage. In order to illustrate the computational advantages of the proposed meta-heuristic algorithm two numerical examples including a 200-bar and a 968-bar double layer grids are presented. The results of SGWA are compared with those of HAS, FA, and GWA and the numerical results indicate that the SGWA converges to better solutions. Finally, it can be concluded that the proposed SGWA can be efficiently employed for design optimization of double layer grids.

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