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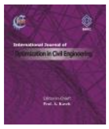

HYBRID ARTIFICIAL PHYSICS OPTIMIZATION AND BIG BANG-BIG CRUNCH ALGORITHM (HPBA) FOR SIZE OPTIMIZATION OF TRUSS STRUCTURES

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ABSTRACT

Over the past decades, several techniques have been employed to improve the applicability of the metaheuristic optimization methods. One of the solutions for improving the capability of metaheuristic methods is the hybrid of algorithms. This study proposes a new optimization algorithm called HPBA which is based on the hybrid of two optimization algorithms; Big Bang-Big Crunch (BB-BC) inspired by the theory of the universe evolution and Artificial Physics Optimization (APO) which is a physical base optimization method. Finally, the performance of the proposed optimization method is compared with the originated methods. Moreover, the performance of the proposed algorithm is evaluated for truss optimization as an applied constrained optimization problem.

Keywords: big bang-big crunch (BB-BC); artificial physics optimization (APO); optimization; metaheuristic methods.

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1. INTRODUCTION

Mathematical programming and metaheuristic techniques are two types of conventional methods that can be used for finding the optimal solution of problem. Due to the gradient-based of the mathematical programming methods the convergence ability of these methods is better than metaheuristic techniques but these methods are applicable only for problems with continuous objective function. This is an essential drawback because there are a lot of problems that have discontinuous objective function especially in engineering fields. As an

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alternative, the metaheuristic methods have not this limitation, these methods can be used for all of the problems with continuous or discontinuous objective function limited by linear and nonlinear constrained.

Metaheuristic optimization methods are inspired by the natural phenomena happen around the world, for instance, genetic algorithm [1, 2] inspired from survival of the fittest theory which was proposed by Darwin, artificial bee colony [3-5] and water strider optimization [6] methods used the behavior of honey bees and water striders in nature, charged system search [7] and gravitational search algorithm [8] have physical base, billiards-inspired optimization algorithm [9] inspired from billiards game and so on. Over the past decades, some of these metaheuristic optimization methods have been modified by various techniques to improve them for more types of problems such as modified versions of the genetic algorithm [10-15], modified teaching-learning optimization method [16] as well as some of them have reasonable performance only for a specific problem like truss optimization [17], steel frame optimization [18], topology optimization problems [19] and so on. It is clear that all of the metaheuristic algorithms and their modified versions do not have enough ability to find the optimum solution of different types of problems. Accordingly, knowing about the capability of each metaheuristic algorithm is useful for choosing an appropriate optimization method and helps to improve the ability of the algorithms by hybridizing them with each other. This study proposes a new optimization algorithm which obtained by hybridizing big bang-big crunch [20] method and artificial physic optimization [21] algorithm then the performance of the proposed hybrid method is compared with originated ones.

1.1 Big bang-big crunch (BB-BC)

Cosmologists present two exciting theories about the universe; theory of universe creation (big bang theory) and theory of universe ending (big crunch). Big bang theory says that the universe created from a single point by incredible energy, big crunch theory says that the universe contracted back to a single point. Big bang-big crunch [20] optimization algorithm inspired by these theories, this method consists of two phases; big bang phase and big crunch Phase.

Big bang phase: similar to the big bang theory, in this phase N initial particles in n-dimensional space are created by Eq. (1).

$$x_j^i = x_{j,min}^i + \text{rand}[0,1]\left(x_{j,max} - x_{j,min}\right) \tag{1}$$

where, $x_{j,max}$ and $x_{j,min}$ are the maximum and the minimum values for the jth variable.

Big Crunch phase: according to the big crunch hypothesis, the universe contracted back to a single point called center of mass. In the big crunch phase this point denoted by $\overrightarrow{x^c}$ that can be computed by using Eq. (2).

$$\overrightarrow{x^c} = \frac{\sum_{i=1}^{N} \frac{1}{f^i} \overrightarrow{x^i}}{\sum_{i=1}^{N} \frac{1}{f^i}} \tag{2}$$

where $\overrightarrow{x^i}$ is the position of the ith particle in search space and f^i is the objective function of the ith particle, then the new particles are generated by using Eq. (3) for next iteration, as follows

$$x^{new} = x^c + \frac{lr}{k} \tag{3}$$

where l is the upper limit of parameter, r is random numbers generated according to a normal distribution with mean zero and standard deviation equal to one and k is the iteration steps.

2.1 Artificial physics optimization algorithm (APO)

The artificial physics optimization (APO) algorithm is a metaheuristic optimization method proposed based on the physicomimetics framework. This method constructs virtual attraction-repulsion force among the particles to move them in search space for computing the optimum solution. APO method consists of four phases including initialization phase, force calculation phase, movement phase and local search phase.

Initialization phase: primary properties of the APO method are determined in this phase, including; number of particles in n-dimensional search space, the position and velocity of each particle and the objective function of each particle. In this phase the position of each particle computed by Eq. (4) and the velocity (v) and the force (F) of each particle are considered zero.

$$x_j^i = x_{j,min}^i + \text{rand}[0,1]\left(x_{j,max} - x_{j,min}\right) \tag{4}$$

where, $x_{j,max}$ and $x_{j,min}$ are the maximum and the minimum of the jth variable, respectively.

Force calculation phase: based on the gravity law, each particle in nature exerts a force on other particles, this force is directly proportional to its mass and inversely proportional to its distance. In APO method the mass of each particle denoted by m_i that can be computed according to Eq. (5), in each iteration.

$$m_i = \begin{cases} K & \text{if } i = \text{best} \\ exp\left(\dfrac{f(x_{best}) - f(x_i)}{max\{f(x_i) - f(x_{best})\}}\right) & \text{if } i \neq \text{best} \end{cases} \tag{5}$$

where K is a positive constant. To determine the force direction between two particles, each one has better objective function attracts the other one, the magnitude of this attraction force can be computed by Eq. (6), in this equation the gravity constant G=1.

$$F_{ij} = \begin{cases} \left(x_j - x_i\right)\dfrac{Gm_i m_j}{\left(x_i - x_j\right)^2} & \text{if } f\left(x_j\right) < f\left(x_i\right) \\ \left(x_i - x_j\right)\dfrac{Gm_i m_j}{\left(x_i - x_j\right)^2} & \text{if } f\left(x_j\right) \geq f\left(x_i\right) \end{cases} \tag{6}$$

Movement phase: movement of each particle in APO method computes according to the second newton law. To this end, the velocity caused by exerted force to each particle should be computed by Eq. (7).

$$v_i\left(t+1\right) = wv_i\left(t\right) + \text{rand}\left[0,1\right]\left(\frac{F_i}{m_i}\right) \tag{7}$$

where the exerted force to each particle F_i and inertia weight of each particle w are computed by Eq. (8) and (9).

$$F_i = \sum_{\substack{i=1 \\ j\neq i}}^{N} F_{ij} \qquad \forall i \neq best \tag{8}$$

$$w = 0.9 - \frac{iteration}{MAXITER} \times 0.5 \tag{9}$$

It can be seen that the variable w is decreased by iteration increasing, in first steps w has the biggest magnitude so APO method has global exploration and in final steps w has smallest magnitude so APO has local exploration. Finally the new position of each particle can be computed by Eq. (10).

$$x_i\left(t+1\right) = x_i\left(t\right) + v_i\left(t+1\right) \tag{10}$$

Local search phase: local search is used to carefully exploit the neighborhood of the best particle. The algorithm of this phase is similar to simulated annealing [22] optimization method. In each iteration, the position of the best particle is the input position of the local search phase, as follows

$$x_{local\ search,k} = x_{best,\ k} - p + \text{rand}\left(0,1\right) \times 2p \qquad ,k = 1,2,\ldots,\text{number of variable} \tag{11}$$

where p is the neighborhood size. If the new particle has better objective function or its

Boltzman equation has bigger than a random value in the interval [0,1], it is replaced. The Boltzman coefficient (*Bl*) is computed by Eq. 12.

$$Bl = exp\left(\frac{f\left(x_{local\ search}\right) - f\left(x_{best}\right)}{\gamma T_k}\right) \tag{12}$$

Where, γ is the temperature descending coefficient that is restricted in the interval [0.3, 0.7] and T_k is the temperature which is started from 100, T_k is decreased by γ coefficient to 10^{-10}. It means that γ determines the number of local search phase done for the best particle in each iteration.

2. HYBRID OPTIMIZATION ALGORITHM

One of the solutions to improve the performance of the optimization methods is hybridizing them with each other. In artificial physics optimization method, the best particle in each iteration is modified by the local search phase. This phase exploits the best solution in the neighborhood of the best particle. In big bang-big crunch optimization method does not exist any phase to explore the neighborhood of the best particle. Accordingly, hybrid of the artificial physics optimization algorithm and big bang-big crunch optimization methods (HPBA) can be a significant incorporation. The pseudo code of this hybrid algorithm is shown in Fig. 1.

```
# Big bang phase
For i=1:number of particles
    For j=1:number of variable
        x_j^i ← x_{j,min}^i + rand[0,1](x_{j,max} - x_{j,min})
    End For
End for
F(X_i): objective function of X_i

# Big crunch
Computed x^c
For i=1:number of particles
    x^new = x^c + lr/k
End for

x_best = arg min{F(X_i), ∀i}

# Local search
T_k = T_0
While T_k > T_f
    For i=1:LISTER
```

```
            For j=1:number of variable
                x_{local search, j} = x_{best, j} - p + rand(0,1) × 2p
            End for
        End for
D= F (x_{local search}) − F(x_{best})
H=Violation (x_{local search})- Violation(x_{best})
If  D<0  & H<=0
x_{best}=  x_{local search}
End if
T_k= γ T_k
End while
```

Figure 1. Pseudo code of proposed optimization method

where x is the position of each particle in search space, x^c is the center of mass which computed by using Eq. (2), T_f is the terminate temperature (usually is equal to 100), γ is temperature descend coefficient and T_0 is the initial temperature (usually is equal to 10^{-10}). The flowchart of the proposed optimization method is shown in Fig. 2.

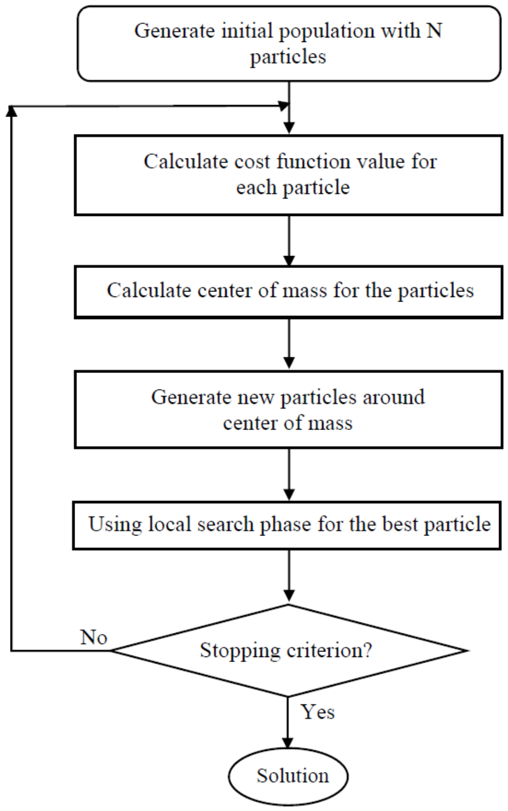

Figure 2. Flowchart of the proposed optimization method

3. VALIDATION OF THE PROPOSED OPTIMIZATION METHOD FOR UNCONSTRAINED PROBLEMS

To investigate the capability of the proposed hybrid optimization method, some unconstrained benchmark functions used in artificial physics optimization and big bang-big crunch methods are selected to analyze the convergence ability of the proposed optimization method.

In the study which was conducted on the artificial physics optimization method two groups of low-dimensional and high-dimensional optimization problems were used. Table 1 shows the properties of the selected functions and number of objective function evaluation to find the optimum solution for APO, BB-BC and proposed optimization methods, more details about these functions are presented in reference [23, 24].

Table 1: The properties of the unconstrained functions selected

Function	n	m	Search range	P	γ	LISTER	APO: function evaluations [21]	BB-BC: function evaluations [22]	HPBA: Function evaluations	Known optimum
Complex	2	10	[-2, 2]	0.01	0.5	10	9500	-	4000	0.0
Davis	2	20	[-100, 100]	1	0.5	10	10000	-	9700	0.0
Himmel-Blau	2	10	[-6, 6]	0.05	0.5	10	8300	-	5500	0.0
Kearfort	4	10	[-3, 10]	0.05	0.3	10	1200	-	700	0.0
Sine Envelope	2	20	[-100, 100]	1	0.3	10	9000	-	3000	0.0
Stenger	2	10	[-1, 4]	0.05	0.5	10	5500	-	2000	0.0
Griewank	2	30	[-100, 100]	1	0.5	10	14500	-	9800	0.0
Tablet	30	20	[-100, 100]	0.5	0.7	3	4.7×10^7	-	3.9×10^7	0.0
Quadric	30	20	[-100, 100]	0.5	0.5	3	3.3×10^7	-	2.9×10^7	0.0
Rosenbrock	30	20	[-50, 50]	0.01	0.5	3	2.4×10^7	-	2.2×10^7	0.0
Griewank	30	20	[-300, 300]	0.5	0.3	3	1.4×10^7	-	1.1×10^7	0.0
Rastrigin	30	20	[-5.12, 5.12]	0.5	0.5	3	1.4×10^7	-	1.2×10^7	0.0
Schaffer's f_7	30	20	[-100, 100]	1	0.3	3	1.4×10^7	-	9.6×10^6	0.0
Sphere	30	10	[-10, 10]	0.01	0.3	3	-	50000	15000	0.0
Rosenbrock	30	10	[-10, 10]	0.01	0.3	3	-	60000	45000	0.0
Step	30	10	[-10, 10]	1	0.3	3	-	1600	1500	0.0
Ellipsoid	30	10	[-10, 10]	0.01	0.3	3	-	45000	20000	0.0
Rastrigin	30	10	[-10, 10]	1	0.3	3	-	300000	200000	0.0
Ackley	30	10	[-10, 10]	1	0.3	3	-	130000	100000	0.0
Shifted Sphere	30	20	[-100, 100]	0.01	0.5	3	1.0×10^7	1.3×10^7	8.1×10^6	-450
Shifted Rosenbrock	30	20	[-100, 100]	0.01	0.5	3	1.5×10^7	1.8×10^7	1.3×10^7	390
Shifted rotated Griewank	30	20	[0, 600]	0.5	0.3	3	1.6×10^7	1.9×10^7	1.5×10^7	-180
Shifted rotated Rastrigin	30	20	[-5, 5]	0.5	0.3	3	1.4×10^7	1.7×10^7	1.2×10^7	-330

In Table 1, n is the space dimension, m is the number of particles and P, T_0, T_f, γ and LISTER are the local search coefficients, the values of T_0 and T_f are 100 and 10^{-10}, respectively. The presented results in Table 1 prove the reasonable performance of the proposed hybrid method. It can be seen that the proposed method estimates the optimum solution with less number of objective function evaluations in comparison with APO and BB-BC methods.

4. VALIDATION OF THE PROPOSED OPTIMIZATION METHOD FOR CONSTRAINED PROBLEMS

Truss optimization is a conventional constrained problem in civil engineering, to evaluate the performance of the proposed hybrid optimization method. To this end, four truss optimization benchmark problems including ten bar- planar truss, seventeen bar- planar truss, twenty five-bar planar truss and seventy two-bar planar truss are selected.

4.1 Ten-bar planar truss

As shown in Fig. 3, a ten-bar planar truss under tow types of load cases is considered; case 1, in which $P_1 = 100$ kips and $P_2 = 0$; and case 2, in which $P_1 = 150$ kips and $P_2 = 50$ kips. For both cases the modulus of elasticity is 10000 ksi and the material density is 0.1 lb/in^3.

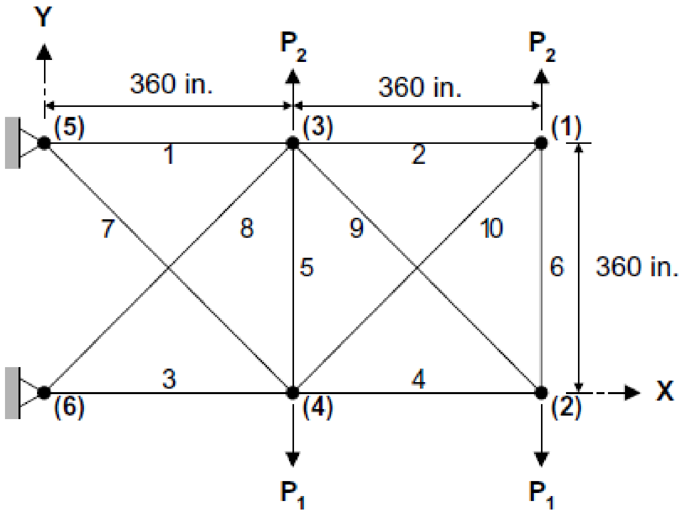

Figure 3. Ten-bar planar truss

Two kinds of limitations are considered for truss optimization; the stress limitation of members is ±25 ksi and the displacement limitation for each node is 2 in. To handle these limitations the constraint handling method which was proposed by Deb [25] is used.

For both considered cases results of the three optimization methods namely; big bang-big crunch (BB-BC), artificial physics optimization (APO) and the proposed optimization method are shown in Table 2. The number of initial particles for each optimization method are 100.

Table 2: Comparison results for ten-bar planar truss

Variables	Case 1			Case 2		
	BB-BC	APO	HPBA	BB-BC	APO	HPBA
A_1	29.685	30.196	30.363	24.9690	23.596	24.004
A_2	0.100	0.100	0.100	0.1000	0.100	0.100
A_3	25.975	24.527	23.803	25.5606	26.662	25.125
A_4	14.998	14.843	14.804	13.6425	14.327	14.332
A_5	0.100	0.100	0.100	0.1000	0.100	0.100
A_6	0.100	0.603	0.556	2.3826	2.009	1.974
A_7	8.269	7.440	7.424	12.5984	12.243	12.515
A_8	20.625	20.956	21.128	13.0452	12.686	12.978
A_9	20.704	21.269	21.472	19.3217	19.760	19.883
A_{10}	0.100	0.100	0.100	0.1000	0.100	0.100
Best Weight (lb)	5084.691	5066.956	5062.010	4697.518	4684.874	4678.112
Average Weight (lb)	5085.421	5067.265	5062.194	4698.253	4685.332	4678.481
Std Dev (lb)	0.611	0.491	0.261	0.577	0.462	0.301
Number of analysis	9000	8500	8000	10500	9000	8000

For artificial physics optimization and proposed methods the coefficients of local search phase namely; P, T_0, T_f, γ and LISTER are 0.05, 100, 1.0e-10, 0.5 and 10, respectively. To evaluate the performance of the proposed method the results of the similar studies which were done on the selected truss problem are used including: Schmit and Farshi [26], Schmit and Miura [27], Venkayya [28], Gellatly and Berke [29], Dobbs and Nelson [30], Khan and Willmert [31] and Kaveh and Talatahari [32]. Results of these studies on case 1 are shown in Table 3.

Table 3: Results of ten-bar planar truss problem (Case 1)

Variables (in^2)	Schmit and Farshi	Venkayya	Gellatly and Berke	Dobbs and Nelson	Khan and Willmert	Kaveh and Talatahari	HPBA
A_1	33.43	30.42	31.35	30.50	30.98	30.307	30.363
A_2	0.10	0.128	0.10	0.10	0.10	0.100	0.100
A_3	24.26	23.41	20.03	23.29	24.17	23.434	23.803
A_4	14.26	14.91	15.60	15.43	14.81	15.505	14.804
A_5	0.10	0.101	0.14	0.10	0.10	0.100	0.100
A_6	0.10	0.101	0.24	0.21	0.406	0.5241	0.556
A_7	8.38	8.696	8.35	7.64	7.547	7.4365	7.424
A_8	20.74	21.08	22.21	20.98	21.05	21.079	21.128
A_9	19.69	21.08	22.06	21.82	20.94	21.229	21.472
A_{10}	0.10	0.186	0.10	0.10	0.10	0.100	0.100
Best Weight (lb)	5089.0	5084.9	5112.0	5080.0	5066.98	5056.56	5062.010

Average Weight (lb)	N/A	N/A	N/A	N/A	N/A	N/A	5062.194
Std Dev (lb)	N/A	N/A	N/A	N/A	N/A	N/A	0.261
Number of analysis	N/A	N/A	N/A	N/A	N/A	N/A	8000

For case 1 the proposed hybrid method has better performance in comparison with some previous studies which were mentioned in literature. Moreover, the proposed algorithm is more accurate than the originated ones. Results of similar studies that were done on case 2 are shown in Table 4.

Table 4: Results of ten-bar planar truss problem (Case 2)

Variables (in^2)	Schmit and Farshi	Venkayya	Dobbs and Nelson	Khan and Willmert	Kaveh and Talatahari	HPBA
A_1	24.29	25.19	25.81	24.72	23.194	24.004
A_2	0.10	0.363	0.10	0.10	0.100	0.100
A_3	23.35	25.42	27.23	26.54	24.585	25.125
A_4	13.66	14.33	16.65	13.22	14.221	14.332
A_5	0.10	0.417	0.10	0.108	0.100	0.100
A_6	1.969	3.144	2.024	4.835	1.969	1.974
A_7	12.67	12.08	12.78	12.66	12.489	12.515
A_8	12.54	14.61	14.22	13.78	12.925	12.978
A_9	21.97	20.26	22.14	18.44	20.952	19.883
A_{10}	0.10	0.513	0.10	0.10	0.101	0.100
Best Weight (lb)	4691	4895.60	5059.7	4792.52	4675.78	4678.112
Average Weight (lb)	N/A	N/A	N/A	N/A	N/A	4678.481
Std Dev (lb)	N/A	N/A	N/A	N/A	N/A	0.301
Number of analysis	N/A	N/A	N/A	N/A	N/A	8000

As shown in Table 4 the proposed hybrid method presents reasonable performance in comparison with studies mentioned in literature. It can be seen that the proposed algorithm has effective performance in comparison with originated ones. For both cases of ten-bar truss optimization problem, Fig. 4 shows the convergence history of the proposed hybrid optimization method. The reported results show that the best optimum solution for both cases of ten-bar truss optimization problem have been found by Kaveh and Talatahari [32].

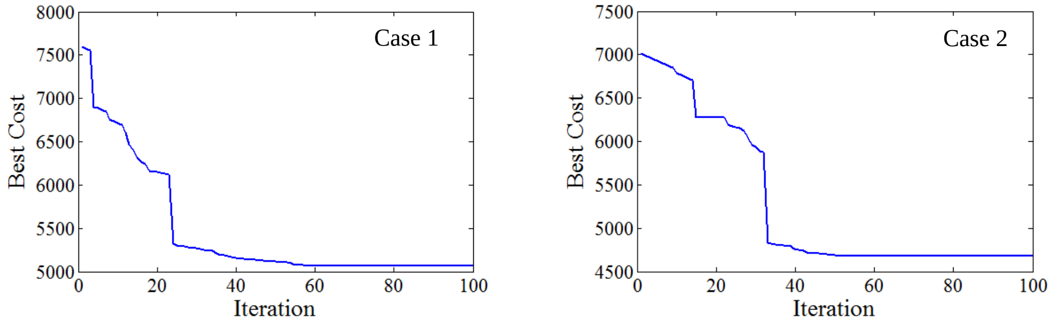

Figure 4. Convergence histories for the optimum solutions of both cases of the ten-bar planar truss

4.2 Seventeen-bar planar truss

The other truss optimization problem is a seventeen-bar planar truss, as shown in Fig. 5. In this problem the material density is 0.268 lb/in^3 and the modules of elasticity is 30000 ksi. For this optimization problem the considered limitations are stress (± 50 ksi) and nodal displacement (±2 in) limitations, the minimum area of cross-sections is 0.1 in^2.

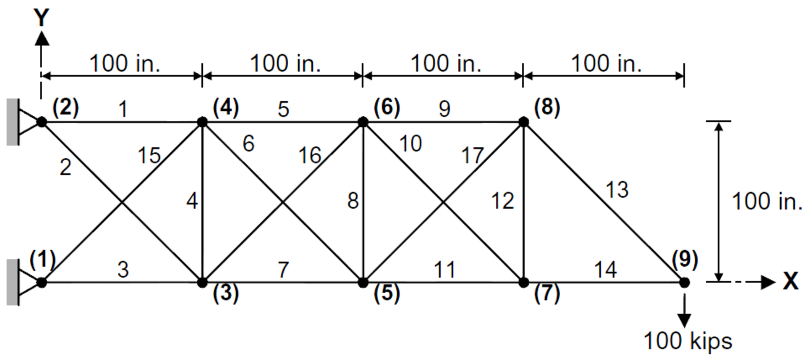

Figure 5. Seventeen-bar planar truss

Table 5 shows the optimum solution of seventeen-bar planar truss obtained by three optimization methods, namely; big bang-big crunch (BB-BC), artificial physics optimization (APO) and hybrid of artificial physics optimization and big bang-big crunch methods (hybrid method). Moreover, some studies were done on seventeen bar planar truss by Adeli and Kumar [33] and Kaveh and Ghazaan [34] that are listed in Table 5.

Table 5: Results for seventeen-bar planar truss

Variables	Adeli and Kumar	Kaveh and Ghazaan	BB-BC	APO	HPBA
A_1	16.029	15.9158	14.4156	16.00	15.80
A_2	0.107	0.1001	0.5150	0.10	0.11
A_3	12.183	12.0762	13.1706	12.28	12.12
A_4	0.110	0.1000	0.1034	0.10	0.10

A_5	8.417	8.0527	8.8999	7.91	8.05
A_6	5.715	5.5611	5.1549	5.52	5.60
A_7	11.331	11.9470	11.4214	12.78	11.97
A_8	0.105	0.1000	0.1101	0.10	0.10
A_9	7.301	7.9425	7.9223	7.45	7.88
A_{10}	0.115	0.1000	0.1782	0.10	0.10
A_{11}	4.046	4.0589	4.4553	3.96	4.07
A_{12}	0.101	0.1000	0.1389	0.10	0.10
A_{13}	5.611	5.6644	5.8455	5.82	5.66
A_{14}	4.046	4.0057	4.1933	3.61	4.05
A_{15}	5.152	5.5565	5.1536	5.63	5.52
A_{16}	0.107	0.1000	0.4065	0.10	0.10
A_{17}	5.286	5.5740	5.4519	5.59	5.61
Best Weight (lb)	2594.42	2581.89	2598.40	2588.98	2582.00
Average Weight (lb)	N/A	2597.11	2599.03	2589.41	2582.74
Std Dev (lb)	N/A	22.41	0.542	0.391	0.214
Number of analysis	N/A	N/A	13000	11000	10500

The coefficients of local search phase consist of; P, T0, Tf, γ and LISTER that are equal to 0.05, 100, 1.0e-10, 0.5 and 10, respectively. As shown in Table 5 and Fig. 6, the obtained results prove the effectiveness and reasonable convergence of the proposed optimization method. Up to now, the best optimum solution for the seventeen bar planar truss has been computed by Kaveh and Ghazaan [34].

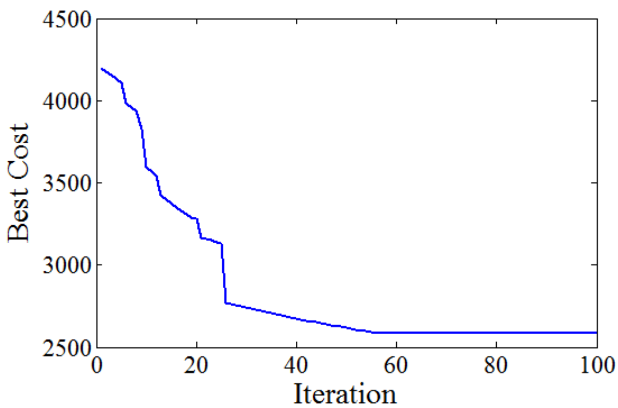

Figure 6. Convergence history for the optimum solution of the Seventeen-bar planar truss

4.3 Twenty five-bar spatial truss

Fig. 7 shows the topology of the twenty five-bar spatial truss with the material density is 0.1 lb/in^3 and modulus of elasticity is 10000 ksi.

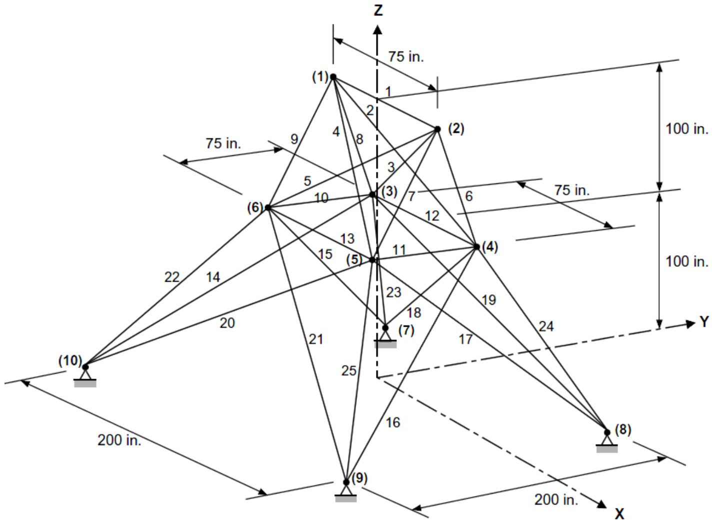

Figure 7. Twenty five-bar spatial truss

The elements of this problem are categorized into eight groups with considering specific displacement and stress limitations. The displacement limitation of each node is 0.35 in each direction and the compressive/tensile stress limitations of each member are shown in Table 6.

Table 6: Stress limitation for the twenty five-bar spatial truss

Element group		Compressive stress limitations ksi	Tensile stress limitations ksi
1	A_1	35.092	40
2	$A_2 \sim A_5$	11.590	40
3	$A_6 \sim A_9$	17.305	40
4	$A_{10} \sim A_{11}$	35.092	40
5	$A_{12} \sim A_{13}$	35.092	40
6	$A_{14} \sim A_{17}$	6.759	40
7	$A_{18} \sim A_{21}$	6.959	40
8	$A_{22} \sim A_{25}$	11.082	40

The minimum and maximum permitted cross section area are 0.01 and 3.4 in^2, respectively. As shown in Table 7, two load conditions are selected for this spatial truss.

Table 7: The applied load conditions to the twenty five-bar spatial truss

Node	Case 1			Case 2		
	P_x kips	P_y kips	P_z kips	P_x kips	P_y kips	P_z kips
1	0.0	20.0	-5.0	1.0	10.0	-5.0
2	0.0	-20.0	-5.0	0.0	10.0	-5.0
3	0.0	0.0	0.0	0.5	0.0	0.0
6	0.0	0.0	0.0	0.5	0.0	0.0

Table 8 compares the performance of the proposed HPBA method with some well-known metaheuristic optimization methods presented by Rajeev and Venkayya [28], Adeli and Kamal [35], Saka [36], Farshi and Ziazi [37] and Kaveh and Talatahari [38].

Table 8: Comparison results for twenty five-bar spatial truss

Variable group	Rajeev and Venkayya	Adeli and Kamal	Saka	Farshi and Ziazi	Kaveh and Talatahari	APO	BB-BC	HPBA
A_1	0.028	0.010	0.010	0.0100	0.010	0.0100	0.3029	0.0100
$A_2 \sim A_5$	1.964	1.986	2.085	1.9981	1.993	2.0136	2.0389	1.9931
$A_6 \sim A_9$	3.081	2.961	2.988	2.9828	3.056	2.9483	3.0208	2.9784
$A_{10} \sim A_{11}$	0.010	0.010	0.010	0.0100	0.010	0.0347	0.0246	0.0100
$A_{12} \sim A_{13}$	0.010	0.010	0.010	0.0100	0.010	0..0100	0.0100	0.0100
$A_{14} \sim A_{17}$	0.693	0.806	0.696	0.6837	0.665	0.7021	0.6276	0.6788
$A_{18} \sim A_{21}$	1.678	1.680	1.670	1.6750	1.642	1.6746	1.6363	1.6798
$A_{22} \sim A_{25}$	2.627	2.530	2.592	2.6668	2.679	2.6570	2.7398	2.6726
Best Weight (lb)	545.45	545.66	545.23	545.37	545.16	545.85	548.58	545.22
Average Weight (lb)	N/A	N/A	N/A	N/A	N/A	546.43	549.31	545.69
Std Dev (lb)	N/A	N/A	N/A	N/A	N/A	0.491	0.553	0.288
Number of analysis	N/A	N/A	N/A	N/A	28850	9000	10000	7000

As shown in Table 8 and Fig. 8, in comparison with originated methods, the proposed hybrid method has acceptable estimation for twenty five-bar spatial truss. Moreover, the results of similar studies prove the reasonable performance and convergence of this method.

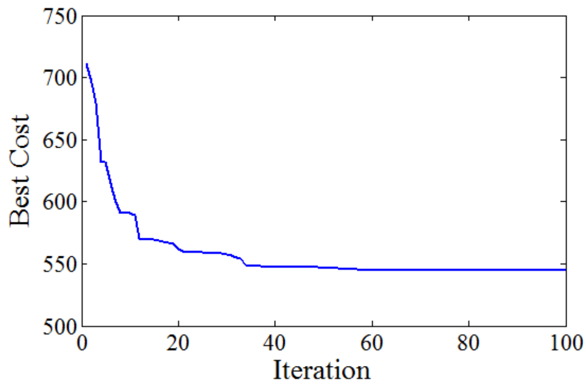

Figure 8. Convergence history for the optimum solution of the twenty five-bar spatial truss

4.4 Seventy two-bar spatial truss

Details of the 72-spatial truss problem are shown in Fig. 9. In this example, the modulus of elasticity is 10000 ksi and the material density is 0.1 lb/in^3.

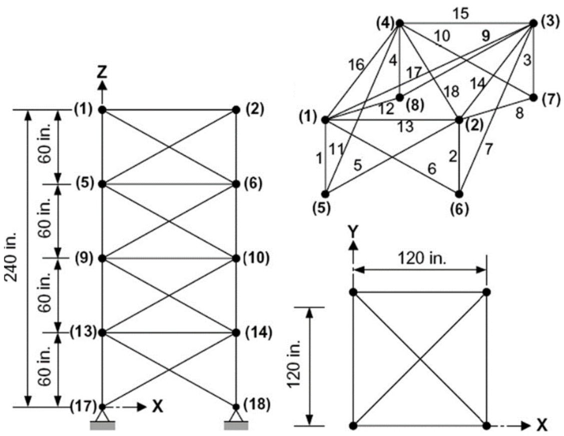

Figure 9. Seventy two-bar spatial truss

For the 72-bar spatial truss problem, the minimum allowable cross-sectional area is 0.1 in^2 and the allowable displacements of uppermost nodes are limited to 0.25 in. The structural members of this spatial truss are sorted into 16 groups with considering specific stress limitation, as detailed in Fig. 9.

Table 9: Stress limitation for the Seventy two-bar spatial truss

Element group		Stress limitations ksi	Element group		Stress limitations ksi
1	$A_1 \sim A_4$	25	9	$A_{37} \sim A_{40}$	25
2	$A_5 \sim A_{12}$	25	10	$A_{41} \sim A_{48}$	25
3	$A_{13} \sim A_{16}$	25	11	$A_{49} \sim A_{52}$	25
4	$A_{17} \sim A_{18}$	25	12	$A_{53} \sim A_{54}$	25
5	$A_{19} \sim A_{22}$	25	13	$A_{55} \sim A_{58}$	25
6	$A_{23} \sim A_{30}$	25	14	$A_{59} \sim A_{66}$	25
7	$A_{31} \sim A_{34}$	25	15	$A_{67} \sim A_{70}$	25
8	$A_{35} \sim A_{36}$	25	16	$A_{71} \sim A_{72}$	25

Table 10 shows the direction of the two load cases applied to the 72-bar spatial truss.

Table 10: The applied load conditions to the Seventy two-bar spatial truss

Node	Case 1			Case 2		
	P_x kips	P_y kips	P_z kips	P_x kips	P_y kips	P_z kips
1	5.0	5.0	-5.0	0.0	0.0	-5.0
2	0.0	0.0	0.0	0.0	0.0	-5.0
3	0.0	0.0	0.0	0.0	0.0	-5.0
4	0.0	0.0	0.0	0.0	0.0	-5.0

Table 11 compares the performance of the proposed hybrid optimization algorithm with the results of some well-known metaheuristic optimization methods presented by Erbatur et al. [39], Camp and Bichon [40], Perez and Behdinan [41] and Kaveh and Talatahari [38].

Table 11: Comparison results for seventy two-bar spatial truss

Variable group	Erbatur et al.	Camp and Bichon	Perez and Behdinan	Kaveh and Talatahari	APO	BB-BC	HPBA
$A_1 \sim A_4$	1.755	1.948	1.7427	1.9042	1.7156	1.7405	1.8035
$A_5 \sim A_{12}$	0.505	0.508	0.5185	0.516	0.5111	0.5396	0.5190
$A_{13} \sim A_{16}$	0.105	0.101	0.1000	0.1000	0.1001	0.1000	0.1000
$A_{17} \sim A_{18}$	0.155	0.102	0.1000	0.1000	0.1000	0.1000	0.1000
$A_{19} \sim A_{22}$	1.155	1.303	1.3079	1.2582	1.2979	1.1876	1.2206
$A_{23} \sim A_{30}$	0.585	0.511	0.5193	0.5035	0.5085	0.5117	0.5128
$A_{31} \sim A_{34}$	0.100	0.101	0.1000	0.1000	0.1000	0.1000	0.1000
$A_{35} \sim A_{36}$	0.100	0.100	0.1000	0.1000	0.1000	0.1000	0.1000
$A_{37} \sim A_{40}$	0.460	0.561	0.5142	0.5178	0.6109	0.6765	0.5399
$A_{41} \sim A_{48}$	0.530	0.492	0.5464	0.5214	0.5295	0.5257	0.5217
$A_{49} \sim A_{52}$	0.120	0.100	0.1000	0.1000	0.1000	0.1028	0.1000
$A_{53} \sim A_{54}$	0.165	0.107	0.1095	0.1007	0.1265	0.1000	0.1000
$A_{55} \sim A_{58}$	0.155	0.156	0.1615	0.1566	0.1536	0.1559	0.1558
$A_{59} \sim A_{66}$	0.535	0.550	0.5092	0.5421	0.5551	0.5250	0.5529
$A_{67} \sim A_{70}$	0.480	0.390	0.4967	0.4132	0.3998	0.3854	0.4363
$A_{71} \sim A_{72}$	0.520	0.592	0.5619	0.5756	0.5736	0.6704	0.5580
Best Weight (lb)	385.76	380.24	381.91	379.66	380.69	381.87	379.85
Average Weight (lb)	N/A	383.16	N/A	381.85	381.12	382.61	380.21
Std Dev (lb)	N/A	3.66	N/A	1.201	0.501	0.607	0.289
Number of analysis	N/A	18500	N/A	13200	15000	18000	11800

As shown in Table 11 and Fig. 10, the results demonstrate a reasonable convergence and accuracy of the proposed HPBA optimization method. “In the study carried by Kaveh and Talatahari [38], a hybrid Big Bang–Big Crunch algorithm was introduced. Although for the spatial truss problems the aforementioned optimization method showed a better accuracy, the proposed HPBA optimization method exhibited a satisfactory accuracy as well as a substantial reduction on the computational cost comparatively.”

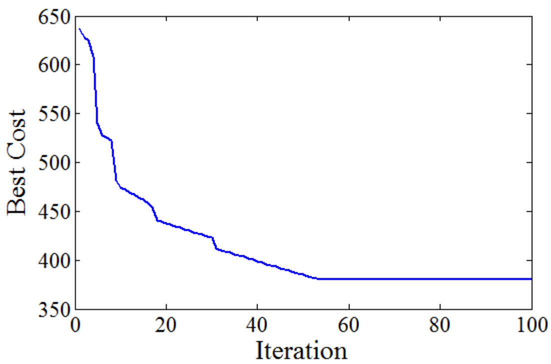

Figure 10. Convergence history for the optimum solution of the Seventy two-bar spatial truss

5. CONCLUSION

This study proposes a new optimization method named HPBA which is a hybrid of big bang-big crunch (BB-BC) and artificial physics optimization (APO) methods. To investigate the performance of the proposed hybrid method for unconstraint problems some benchmark functions which were used for evaluation of APO and BB-BC algorithms are selected, these functions consist of many low-dimensional and high-dimensional functions. The computed results show that the proposed hybrid optimization method finds the optimum solution with lower number of objective function evolution in comparison with APO and BB-BC methods.

To evaluate the performance of the proposed hybrid method for constraint problems, truss optimization is selected as a conventional structure in civil engineering. Three benchmark truss optimization problems with various constraints under different types of loads are selected; ten-bar planar truss with different type of loads, seventeen-bar planar truss, twenty five-bar spatial truss and seventy two-bar spatial truss, for all of them the optimum solutions which are obtained by the proposed hybrid method are more accurate in comparison with BB-BC and APO optimization methods.

As shown for constraint and unconstraint problems the proposed hybrid method which is a hybrid of BB-BC and APO methods has more ability to find the optimum solution among the feasible search space in comparison with originated methods.

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