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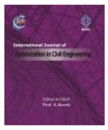

# SEQUENTIAL PENALTY HANDLING TECHNIQUES FOR SIZING DESIGN OF PIN-JOINTED STRUCTURES BY OBSERVER-TEACHER-LEARNER-BASED OPTIMIZATION

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## ABSTRACT

Despite comprehensive literature works on developing fitness-based optimization algorithms, their performance is yet challenged by constraint handling in various engineering tasks. The present study, concerns the widely-used external penalty technique for sizing design of pin-jointed structures. Observer-teacher-learner-based optimization is employed here since previously addressed by a number of investigators as a powerful meta-heuristic algorithm. Several cases of penalty handling techniques are offered and studied using either maximum or summation of constraint violations as well as their combinations. Consequently, the most successive sequence, is identified for the treated continuous and discrete structural examples. Such a dynamic constraint handling is an affordable generalized solution for structural sizing design by iterative population-based algorithms.

**Keywords:** constraint handling; meta-heuristic algorithm; optimal structural design; sizing design; truss structures.

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## 1. INTRODUCTION

Most of the real-world problems are distinguished with constraints that must be satisfied prior to accept the arbitrary or optimal designs. Structural problems are a common set among them that usually include several inequality constraints [1,2]. Up to date, there exists compromise between computational efficiency in cost reduction and feasibility preservation [3–6]. Therefore, developing efficient constraint handling techniques is still an active field

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of research.

Early years of such a research activity is identified by explorative attempts to develop various ideas [7]. They can be categorized within: special operators, decoders and mappers, separation of objective function and constraints or integrating them via penalty functions [7]. Repair-operators and decoders may be suited to some combinatorial problems. However, penalty approaches are among the well-accepted and widely used techniques for numerical engineering applications. They are themselves divided into sub-categories including: *static penalty, adaptive penalty, dynamic penalty, annealing and co-evolutionary penalties*. Another approach is to discard infeasible solutions that arise during optimization; it is known as *death penalty*.

Meta-heuristic algorithms include population-based methods that essentially deal with a single objective function to be sampled over the search space [8]. So they mostly use single penalty functions when dealing with constraint engineering problems. A number of recent algorithms in this category can be referred to as Water Evaporation Optimization [9], Heat Transfer Search [10], Switching Teams Algorithm [11], Plasma Generation Algorithm [12], Social Network Search [13], Remora Optimization Algorithm [14], African Vultures Optimization Algorithm [15], Blood Coagulation Algorithm [16], Escaping Bird Search [17] and Black Hole Mechanics Optimization [18].

Penalty approaches usually maintain some problem-dependent factors to be specified for optimization. In order to better study their effects, it is desired to deal with the algorithms that have minimal number of control parameters to be tuned. Here, we consider *Observer-Teacher-Learner Based Optimization* (OTLBO) as an enhanced variant of TLBO that have already received attention in civil engineering problems [19–22].

The most popular approach in constraint handling; i.e. linear external penalty is concerned here-in-after. In this regard, two basic constraint violation functions are distinguished and performance of their offered consequences is compared through a comprehensive study using discrete and continuous structural sizing examples.

## 2. OPTIMIZATION ALGORITHM

*Teaching-Learning-Based Optimization* (TLBO) [23,24] is a popular method with wide applications in several engineering fields [25–28]. Such a meta-heuristic algorithm simulates the knowledge growth in a classroom via distinct phases; the first is improving the mean level of the students' grades by a teacher while the second mimics learning via interaction between students themselves.

As an enhanced variant of TLBO, *Observer-Teacher-Learner-Based Optimization*, OTLBO, has been introduced by embedding an observer-phase to the algorithm [16]. It applies extra memory exploitation for more effective search. OTLBO has already been applied to various engineering problems including ground motion scaling [16], optimal sizing of structures [17], active and semi-active control of high-rise buildings [18] and prediction of environmental phenomena by embedded machine learning [19]. Consequently, OTLBO is considered as one of the powerful methods in the class of parameter-less algorithms. The steps of OTLBO algorithm are reviewed as follows:

- *Initiation*; generate a random population of $N_p$ classmates between their lower and upper limits $x_j^L$ and $x_j^U$ in any j[th] subject, respectively.

$$\begin{aligned} &X_j^i = q(X_j^L + rand \times (X_j^U - X_j^L)) \\ &i \in \{1,2,...,N_p\}, \quad j \in \{1,2,...,N_d\} \end{aligned} \tag{1}$$

$q(.)$ denotes a function to preserve upper and lower bounds and to round variables for discrete problems. $rand$ is a random generator.

- Rank the classmates in *Population* (Pop.) based on their marks. Identify the global best as the teacher:

$$Pop = \begin{bmatrix} x_1^1 & x_2^1 & ... & x_{Nd}^1 \\ x_1^2 & x_2^2 & ... & x_{Nd}^2 \\ & & ... & \\ x_1^{Np} & x_2^{Np} & ... & x_{Nd}^{Np} \end{bmatrix} \tag{2}$$

- For any $i^{th}$ student, perform either *Teacher-phase* or *Observer-phase* by equal chance:
- ○ *Teaching phase*: construct a candidate classmate obeying the teacher as:

$$X^{new,i} = q(X^{old,i} + rand \times (X^{Teacher} - T_f X^{Mean})) \tag{3}$$

where as $T_f$ denotes a teaching factor that randomly switches between either 1 or 2 .

- ○ *Observer phase*: For any $i^{th}$ student, construct the following candidate solution by exploiting the memory for every its $j^{th}$ component:

$$X_j^{new,i} = X_j^{Exploited} \tag{4}$$

- ○ Greedy selection: replace $X^i$ with $X^{new,i}$ if $X^{new,i}$ is better than it.
- *Learning phase*: perform interactive actions between couples of classmates:
- ○ Randomly select a couple of distinct classmates number *i* and *j*.
- ○ Determine $X^{new,i}$ by:

$$\begin{aligned} &X^{new,i} = q(X^{old,i} + rand \times (X^j - X^i)) \qquad \text{if} \quad Fit(X^j) > Fit(X^i) \\ &X^{new,i} = q(X^{old,i} + rand \times (X^i - X^j)) \qquad otherwise \end{aligned} \tag{5}$$

- ○ Greedy selection: replace $X^i$ with $X^{new,i}$ if $X^{new,i}$ is better than it

- Repeat the aforementioned phases for the prescribed number of ierations, $t_{\max}$ .

A brief main code of OTLBO is given in the Appendix, employing general subroutines that interested user can adapt for population-based methods.

## 3. SIZING PROBLEM FORMULATION

A vast category of structural optimization problems, is characterized by minimizing the amount of construction material subject to behavior constraints on structural responses as well as the simple bounds on member sizes. Satisfying the system of equilibrium equations, is a crucial requirement that is implicitly applied via structural analysis rather than explicitly arising in the problem formulation. The structural sizing problem can be formulated to minimize structural weight, *W*, as:

$$\begin{aligned} &\min W(X) \\ &\text{s.t.} \\ &x^L \le x_i \le x^U \\ &g_j(X) \le 0 \quad , \quad j = 1,\dots,N_c \end{aligned} \tag{6}$$

The side constraints are commonly satisfied by a fly-to-boundary technique for enforcing the size of member sections to fall within their lower and upper bounds; i.e. $x_i^L$ and $x_i^U$ , respectively.

In the other hand, structural design codes require that the stress and/or deflection responses must fall below their available limits according to the applied loading and the corresponding analysis procedure. This last set, constitutes the remained constraints to be satisfied during structural optimization. They are generally expressed in the standard form of inequality equations as:

$$g_j(X) \le 0 \quad , \quad j = 1,\dots,N_c \tag{7}$$

where $N_c$ stands for the total number of behavior constraints.

## 4. THE PROPOSED CONSTRAINT HANDLING TECHNIQUES

Suppose that violation of every behavior constraint is expressed as:

$$G_j(X) = \begin{cases} 0 & \text{if } g_j(X) \le 0 \\ g_j(X) & \text{otherwise} \end{cases} \tag{8}$$

It can be further assessed by either supermom function $V_{sup}$ :

$$\mathrm{V}_{sup}(X) = \max_{j} G_j(X) \tag{9}$$

or summation $V_{sum}$ :

$$\mathrm{V}_{sum}(X) = \sum_{j} G_j(X) \tag{10}$$

Either case can be implemented in form of the following pseudo-unconstrained cost function, $\varphi$ to be minimized during the optimization process:

$$Minimize\ \varphi(X) = f(X) \times [1 + k_P \times V(X)] \tag{11}$$

where $f$ stands for the structural weight as the row cost function and $k_p$ denotes the corresponding penalty factor. In the above equations; $X$ is the corresponding design vector denoting the member-group section-areas for the structural sizing problem.

Optimization of the problem using $V_{sum}$ will also lower $V_{sup}$ while the reverse is not necessarily true. Despite the summation, the supermom function does not reveal information about total violation of constraints. In the other hand, $V_{sup}$ exerts less penalty than $V_{sum}$ on individuals that allow more members to approach the active state of their corresponding stress/deflection constraints.

Table 1: Definition of the applied penalty-handling types

| **PT** | Phase 1 | Phase 2 | Phase 3 |
|---|---|---|---|
| **1** | $V_{sum}$ | $V_{sum}$ | $V_{sum}$ |
| **2** | $V_{sup}$ | $V_{sup}$ | $V_{sup}$ |
| **3** | $V_{sup}$ | $V_{sum}$ | $V_{sum}$ |
| **4** | $V_{sup}$ | $V_{sup}$ | $V_{sum}$ |
| **5** | $V_{sum}$ | $V_{sup}$ | $V_{sup}$ |
| **6** | $V_{sum}$ | $V_{sum}$ | $V_{sup}$ |
| **7** | Random | Random | Random |

PT: Penalty-handling Type

A more comprehensive study is needed to decide which case is better during optimal design of a specific structural problem. In this regard, we offer a general framework to allow implementation of either cases or sequential combination of them. Consider a typical optimization algorithm that iterates from $t=1$ up to $t_{\max}$ in its main loop. Let's subdivide total iterations into the following three phases:

Phase 1: from 1 to $0.25t_{\max}$

Phase 2: from $0.25t_{\max}$ to $0.75t_{\max}$

Phase 3: from $0.75t_{\max}$ to $t_{\max}$

By altering the choice of violation function among such phases, we offer a number of *Penalty-handling Types* (PT's) as given in Table 1. Note that either $V_{sum}$ or $V_{sup}$ is implemented in each phase; except for PT-7 that selection of $V_{sum}$ or $V_{sup}$ is randomly done by equal chance at every iteration.

## 5. NUMERICAL EXPERIMENTS

Every PT is evaluated on continuous and discrete examples of spatial pin-jointed structures. In each example, four different penalty factors are applied within each of 48 independent trail runs. OTLBO is used as the optimization algorithm with minimal control parameters of $t_{\max}$ and the population size: $N_p$. Table 2 briefs the applied parameters for the treated examples. Consequently, 28 different cases of constraint handling is implemented for each example as listed in Table 3.

Table 2: Applied parameters for the treated examples

| **Example** | $N_p$ | $t_{\max}$ | $N_{runs}$ | $N_g$ | $N_{Sect}$ | $N_c$ | $k_p^{(1)}$ | $k_p^{(2)}$ | $k_p^{(3)}$ | $k_p^{(4)}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **Tower Truss** | 60 | 600 | 48 | 32 | 247 | 1623 | 70 | 80 | 90 | 100 |
| **Helipad Truss** | 60 | 600 | 48 | 9 | N.A. | 1983 | 50 | 60 | 70 | 80 |

N.A.: Not Available

Table 3: List of penalty handling types and implemented penalty factors

| $k_p$ | CN | PT | CN | PT | CN | PT | CN | PT | CN | PT | CN | PT | CN | PT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $k_p^{(1)}$ | **1** | 1 | **5** | 2 | **9** | 3 | **13** | 4 | **17** | 5 | **21** | 6 | **25** | 7 |
| $k_p^{(2)}$ | **2** | 1 | **6** | 2 | **10** | 3 | **14** | 4 | **18** | 5 | **22** | 6 | **26** | 7 |
| $k_p^{(3)}$ | **3** | 1 | **7** | 2 | **11** | 3 | **15** | 4 | **19** | 5 | **23** | 6 | **27** | 7 |
| $k_p^{(4)}$ | **4** | 1 | **8** | 2 | **12** | 3 | **16** | 4 | **20** | 5 | **2 4** | 6 | **28** | 7 |

CN: Case Number, PT: Penalty-handling Type

Cardinality of a discrete example with $N_g$ member groups and $N_{Sect}$ available sections, will be $C_r = (N_{Sect})^{N_g}$. However, $N_{Sect}$ is *Not Applicable* (N.A.) for continuous examples as their member properties can get infinite values within their lower-to-upper bounds. Taking into account that in OTLBO the number of function evaluations for each individual at any iteration is $FE_{ii} = 2$ [17,19], 193,536,000 structural analyses have totally been performed on a multi-processor platform to support reliable results for the present comprehensive study.

In order to have an insight on resulted feasibility of optimal designs; a statistical measure is defined as follows:

$$PFD = 100 \times \frac{N_F}{N_T} \tag{12}$$

PFD stands for *Percentage of Feasible Designs* in an experinment being the ratio of the feasible designs quantity; $N_F$ over total number of samples; $N_T$ . Tracing such a metric declares how an explicit constraint handling technique affects feasibility of the resulted optimal designs over the considered set of trial runs.

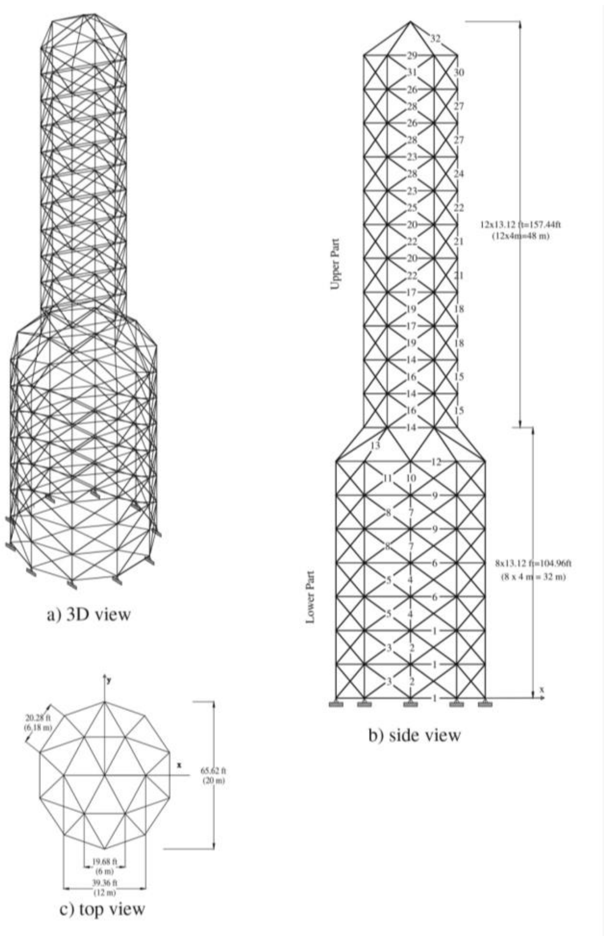



Figure 1. 582-bar tower truss

*5.1 582-bar tower truss*

The 582-bar space tower is treated here as a large-scale problem with discrete variables. It has already been studied by several investigators including [29–31]. As shown in Fig. 1, the members are divided into 32 groups. The modulus of elasticity is 29,000 *ksi* (200 *GPa*) and the material density is 0.283 $lb/in^3$ (7,833.4 $kg/m^3$) with the yield stress of 36 *ksi*. The stress and slenderness constraints are considered during optimization due to AISC-ASD89 regulations [32] as:

$$\sigma_{tension}^{allowable} = 0.6F_y \tag{13}$$

$$\sigma_{compression}^{allowable} = \begin{cases} \dfrac{12\pi^2 E}{23\lambda^2} & \text{for } \dfrac{\lambda}{C_c} \geq 1 \\ (1-\dfrac{\lambda^2}{2C_c^{\,2}})F_y \Big/ (\dfrac{5}{3}+\dfrac{3}{8}\dfrac{\lambda}{C_c}-\dfrac{\lambda^3}{8C_c^{\,3}}) & \text{for } \dfrac{\lambda}{C_c} < 1 \end{cases} \tag{14}$$

$$C_c = \sqrt{2\pi^2 E \Big/ F_y}$$

where λ denotes the slenderness ratio of the corresponding member. It is calculated dividing the effective length by section gyration radius. In addition, every nodal displacement is limited to 3.15 *in* (8 *cm*). As a service constraint, the slenderness ratio is limited to 300 for tension members and to 200 for compression members. Horizontal lateral load of 1.12 *kips* (5.0 *kN*), is applied at each free node in both x and y directions. The upper-part nodes undergo downward vertical loads of 6.74 *kips* (30 *kN*) where 3.37 *kips* (15 *kN*) is applied at every lower-part node. The cross sections are selected from a discrete list of W-shaped AISC profiles; so that the cardinality of search space in this example is as large as $C_r \approx 3.7\times10^{76}$.

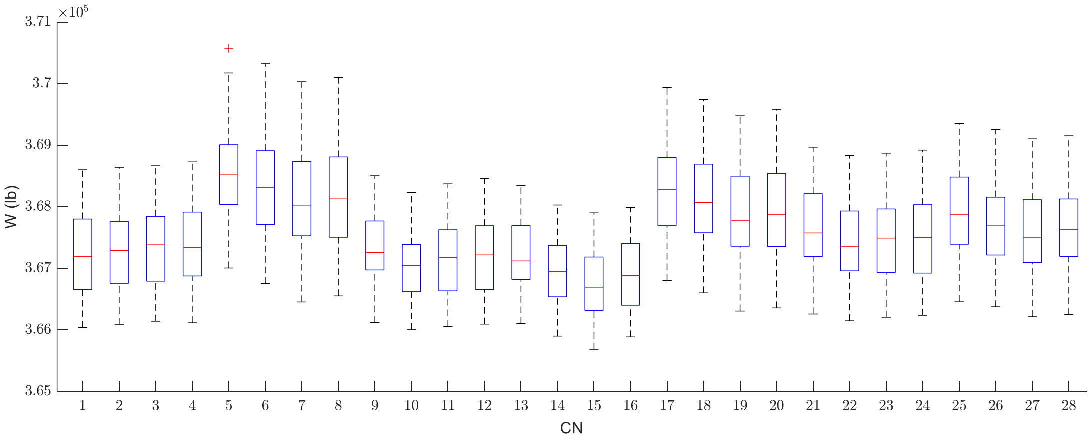



Figure 2. Statistical representation of the results for 582-bar tower truss

Table 4: Comparison of optimal designs for 582-bar tower truss

| Variable | ABC Case 1[30] | ABC Case 2[30] | Present work |
|---|---|---|---|
| $X_1$ | W8X21 | W8X21 | W8X21 |
| $X_2$ | W18X86 | W10X17 | W14X82 |
| $X_3$ | W8X24 | W8X24 | W8X24 |
| $X_4$ | W10X60 | W14X61 | W10X60 |
| $X_5$ | W8X24 | W8X24 | W8X24 |
| $X_6$ | W8X21 | W8X21 | W8X21 |
| $X_7$ | W10X49 | W10X60 | W10X49 |
| $X_8$ | W8X24 | W8X24 | W8X24 |
| $X_9$ | W8X21 | W8X21 | W8X21 |
| $X_{10}$ | W12X53 | W10X49 | W10X39 |
| $X_{11}$ | W8X24 | W8X24 | W8X24 |
| $X_{12}$ | W21X62 | W10X68 | W10X68 |
| $X_{13}$ | W27X84 | W18X76 | W12X79 |
| $X_{14}$ | W10X45 | W14X48 | W8X48 |
| $X_{15}$ | W10X84 | W10X77 | W24X76 |
| $X_{16}$ | W8X31 | W8X31 | W8X31 |
| $X_{17}$ | W8X21 | W8X21 | W8X21 |
| $X_{18}$ | W12X53 | W21X62 | W14X61 |
| $X_{19}$ | W8X24 | W8X24 | W8X24 |
| $X_{20}$ | W10X22 | W8X21 | W8X21 |
| $X_{21}$ | W16X36 | W14X43 | W8X40 |
| $X_{22}$ | W8X24 | W8X24 | W8X24 |
| $X_{23}$ | W8X21 | W8X21 | W8X21 |
| $X_{24}$ | W10X22 | W8X24 | W8X24 |
| $X_{25}$ | W6X25 | W8X24 | W8X24 |
| $X_{26}$ | W8X21 | W8X21 | W8X21 |
| $X_{27}$ | W8X21 | W8X21 | W8X21 |
| $X_{28}$ | W8X24 | W8X24 | W8X24 |
| $X_{29}$ | W8X21 | W8X21 | W8X21 |
| $X_{30}$ | W10X22 | W8X21 | W8X21 |
| $X_{31}$ | W8X24 | W8X24 | W8X24 |
| $X_{32}$ | W6X25 | W8X24 | W8X24 |
| Best W (*lb*) | 368484.1 | 365906.3 | 365686.7 |
| Mean W (*lb*) | 370178.6 | 366088.4 | 366747.0 |
| SD | - | - | 629.80 |

SD: Standard Deviation

Fig. 2 shows meaningful difference between distinct sets of PT's regarding optimality of designs. It is observed that sequential applications of $V_{sum}$ and $V_{sup}$ can reveal lower optimal weights than mere application of each; regarding the best or mean samples. The best result has been obtained in CN-14 as a subset of PT-4; i.e. starting 75% of iterations with $V_{sup}$ followed by $V_{sum}$ up to the last iteration. Results of applying PT-5 or PT-6 (starting with $V_{sum}$ and continuing with $V_{sup}$) are not as good as those by PT-4 and even PT-3. In a general view, PT-7 has not better a performance than PT-1 and PT-6; but stands superior to PT-2 and PT-5 in this example.

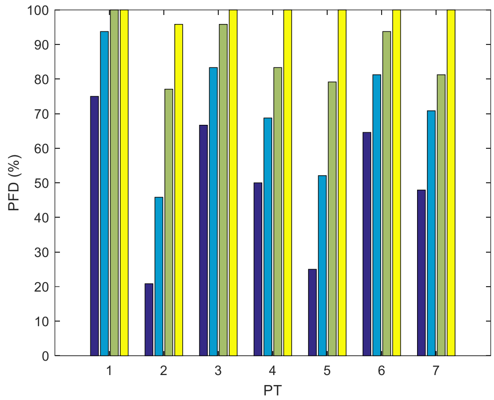



Figure 3. Percentage of feasible designs in the results of 582-bar truss optimization

Another issue to study, is variation of feasibility ratio with changing the penalty handling technique. Fig. 3 declares that for each PT, increasing the penalty factor can increase PFD; however, it is not a general case in comparison between different PT's. Note that in PT-1, both $k_p^{(3)}$ and $k_p^{(4)}$ have led to full PFD. It shows increasing the penalty factor over a threshold ($k_p^{(2)}$ ~80 in this example) has behaved such as applying the death penalty that allows no infeasible design.

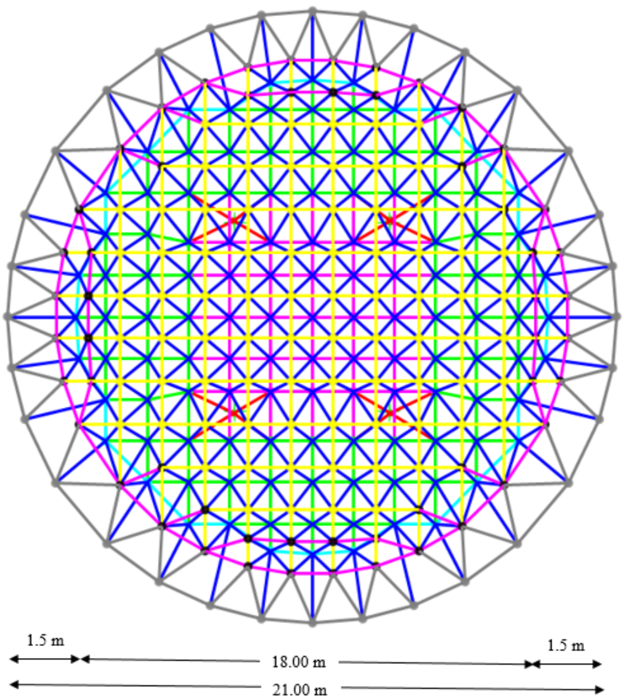
Table 5: Percentage of feasible designs in optimization of 582-bar tower truss

| CN | PFD | CN | PFD | CN | PFD | CN | PFD | CN | PFD | CN | PFD | CN | PFD |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 75.0 | **5** | 20.8 | **9** | 66.7 | **13** | 50.0 | **17** | 25.0 | **21** | 64.6 | **25** | 47.9 |
| **2** | 93.7 | **6** | 45.8 | **10** | 83.3 | **14** | 68.7 | **18** | 52.1 | **22** | 81.3 | **26** | 70.8 |
| **3** | 100.0 | **7** | 77.1 | **11** | 95.8 | **15** | 83.3 | **19** | 79.2 | **23** | 93.7 | **27** | 81.3 |
| **4** | 100.0 | **8** | 95.8 | **12** | 100.0 | **16** | 100.0 | **20** | 100.0 | **2 4** | 100.0 | **28** | 100.0 |
| **SD** | 11.8 | **SD** | 33.2 | **SD** | 15.0 | **SD** | 21.3 | **SD** | 32.6 | **SD** | 15.6 | **SD** | 21.7 |

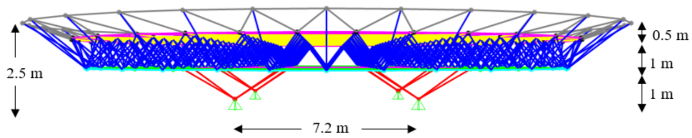
SD: Standard Deviation

The optimal weight of 582-bar tower is obtained 365686.7 *lb*; that is superior to other literature works reported in Table 4. Fig. 3 reveals that such a least optimal weight belongs to CN-15 in PT-4. It is worth notifying that the corresponding PFD values in PT-4, have fallen within 70% to 100% .

It can also be realized from Table 5 that standard deviation of PFD varies depending on the applied PT. Employing identical set of penalty factors; the most diverse variation of PFD is observed in PT-2 and PT-5. It is while PT-3 and PT-4 has not shown such diverse PFD's over the applied penalty factors, in this example.




(a)




(b)

Figure 4. 1104-bar helipad truss: (a) top view, (b) side view [33]

*5.2 1104-bar helipad truss*

This example has been introduced by Shahrouzi and Salehi [33] as a continuous real-world problem (Fig. 4). Truss members are divided into 9 groups and their section areas vary between 10 $cm^2$ and 100 $cm^2$. Material density, modulus of elasticity and yield stress are $\rho$ =7850 $kg/m^3$ , E=203.9 *GPa* and $F_y$=253.1 *MPa*, respectively. Uniform gravitational load of 300 $kgf/m^2$ is exerted on the top level of the helipad. Besides, concentrated load of 350 *kgf* is applied at each of four central nodes. Stress constraints are applied due to AISC-ASD89 design code [32]. The nodal displacement in every orthogonal direction is limited to 5 *cm*.

According to Table 6, OTLBO in the present work has obtained the best weight of helipad among the others; that is 32588.9 *kg*. According to Fig. 5, such a result belongs to

PT-4 (CN-14). It is better than 32999.8 kg as the best of previous works that applied pure $V_{sum}$ [33]. In this example, PT-4 has been superior to the others regarding both the best and mean optimal weight. PT-3 is on the second rank while PT-2 and PT-5 have not been as good as the others. Again PT-7 with random selection of either case, has stood on a moderate rank. Fig. 5 also declares that starting with $V_{sup}$ and continuing with $V_{sum}$ leads to lower costs in the treated structural problem.

Table 6: Comparison of optimal designs for 1104-bar helipad truss

| Variable | BA [33] | LAPO [33] | ICA [33] | ICLBO [33] | Present work |
|---|---|---|---|---|---|
| $A_1$ (cm$^2$) | 15.09 | 34.88 | 34.88 | 40.03 | 12.55 |
| $A_2$ | 10.13 | 16.36 | 16.36 | 19.27 | 13.79 |
| $A_3$ | 19.57 | 10.00 | 10.00 | 10.00 | 10.00 |
| $A_4$ | 23.75 | 22.87 | 22.87 | 23.21 | 27.30 |
| $A_5$ | 35.56 | 42.68 | 42.68 | 29.15 | 32.50 |
| $A_6$ | 45.93 | 22.6 | 22.6 | 20.45 | 17.30 |
| $A_7$ | 49.31 | 41.51 | 41.51 | 16.94 | 29.94 |
| $A_8$ | 14.49 | 46.19 | 46.19 | 27.43 | 29.16 |
| $A_9$ | 52.54 | 66.17 | 66.17 | 82.82 | 50.96 |
| Best W (*kg*) | 35520.62 | 36440.05 | 36440.05 | 32999.8 | 32588.9 |
| Mean W (*kg*) | 47511.46 | 38020.01 | 38020.01 | 36781.07 | 33446.77 |
| SD | - | - | - | - | 591.47 |

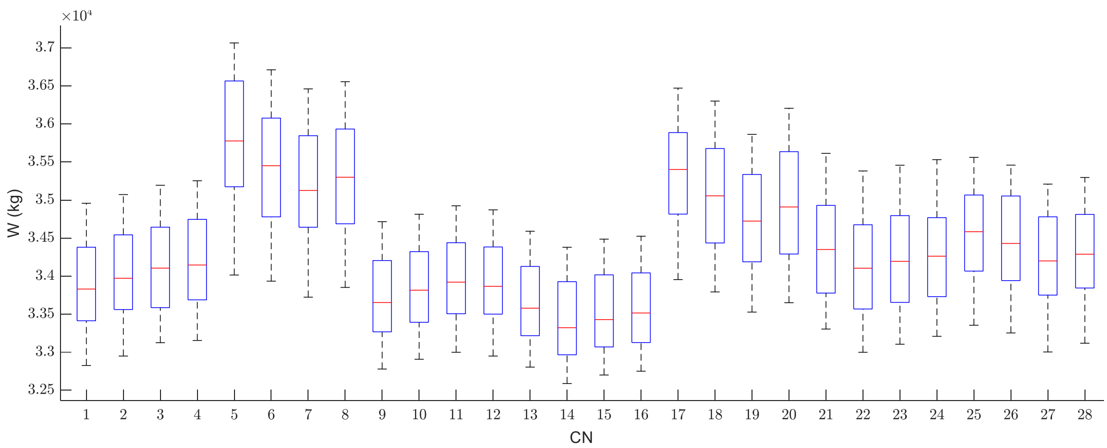



Figure 5. Statistical representation of the results for 1104-bar helipad truss

It can also be noticed that the best penalty factor differs among different PT's. Such a first rank of optimal cost belongs to $k_p^{(1)}$ in the PT-1 and PT-3, $k_p^{(2)}$ in PT-4 and PT-6 and $k_p^{(3)}$ in the PT-2, PT-5 and PT-7. It is while $k_p^{(4)}$ has not taken the first rank in any PT. Fig. 6 reveals that applying such a high penalty factor has resulted in full feasibility of optimal designs and acts such as the death penalty technique. It again confirms that penalty factors below a large threshold of full feasibility, can provide better optimality in the resulted cost values. In another word, it is not expected that the treated penalty methods, simultaneously provide full feasibility and the best optimality.

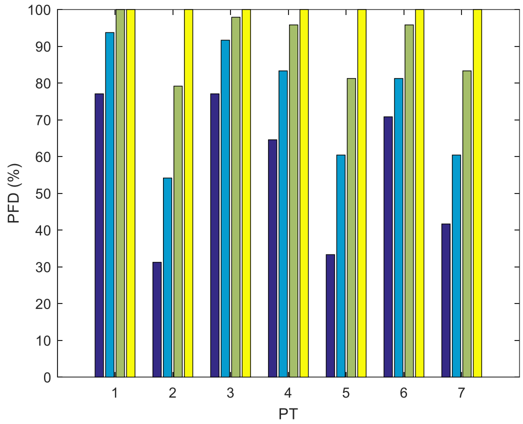



Figure 6. Percentage of feasible designs in the results of 1104-bar truss optimization

In addition to Fig. 6, Table 7 gives an insight to compare the deviation of PFD’s among different PT’s, in this example. It is observed that applying pure $V_{sup}$ has led to more diverse PFD’s. In contrary, the least standard deviation of PFD’s belongs to PT-3 that starts with $V_{sup}$ and continues with $V_{sum}$ .

Table 7: Percentage of feasible designs in optimization of 1104-bar helipad truss

| CN | PFD | CN | PFD | CN | PFD | CN | PFD | CN | PFD | CN | PFD | CN | PFD |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 77.1 | **5** | 31.2 | **9** | 77.1 | **13** | 64.6 | **17** | 33.3 | **21** | 70.8 | **25** | 41.7 |
| **2** | 93.7 | **6** | 54.2 | **10** | 91.7 | **14** | 83.3 | **18** | 60.4 | **22** | 81.2 | **26** | 60.4 |
| **3** | 100.0 | **7** | 79.2 | **11** | 97.9 | **15** | 95.8 | **19** | 81.2 | **23** | 95.8 | **27** | 83.3 |
| **4** | 100.0 | **8** | 100.0 | **12** | 100.0 | **16** | 100.0 | **20** | 100.0 | **24** | 100.0 | **28** | 100.0 |
| **SD** | 10.8 | **SD** | 29.9 | **SD** | 10.3 | **SD** | 15.9 | **SD** | 28.6 | **SD** | 13.4 | **SD** | 25.6 |

SD: Standard Deviation

## 6. CONCLUSION

The present work, distinguished two major types of violation functions for inequality constraints that are common in structural sizing problems. Their pure or combined application via seven penalty-handling types were studied. A number of concluding remarks are briefed as follows; based on the obtained statistical results.

Although application of large penalty factors preserves full feasibility over different trial runs, it affects the search refinement of the optimization algorithm and does not guaranty overpassing local optima to capture the best optimal cost. The proper penalty factor to capture minimal cost, is usually less than a value that enforces all designs to be feasible. It depends not only on the problem but also on the type of penalty function for constraint handling.

Considering the treated cases, it was observed that sequential application of $V_{sum}$ and $V_{sup}$ reveals better feasible results than applying only one of them. It was also declared that starting with $V_{sum}$ and continuing the remainder iterations with $V_{sup}$ can reveal the most desired result; i.e. the least-weight feasible design.

PFD is defined and traced as a measure for probability of achieving feasible designs via independent runs of the algorithm. It was observed that the most successive sequential penalty-handling types, brought about relatively lower deviation in PFD than the other PT's. In another word, they can provide superior optimality; however, with partial feasibility ratio of optimal designs over different runs. According to this study, the best results of structural sizing design were achieved when the staring part of the search (with $V_{sup}$) was two-third of total iterations ending with $V_{sum}$; i.e. by applying PT-4. The corresponding PFD was about 83% in the treated examples. Although such a value is below full PFD; the case is affordable in an overall view concerning both feasibility and optimality. A future scope of research, is to study nonlinear penalty-included as well as penalty-free techniques in optimization of engineering problems.

## APPENDIX

A brief MATLAB function for OTLBO

```
% Observer-Teacher-Learner-Based Optimization by Mohsen Shahrouzi, 2016
function  [xOpt,FitOpt]=OTLBO(NumItrs,PopSize)
% Initiation
Initiate4Opt;
% main loop
for Iter=2:NumItrs
   for i=1:PopSize
      if rand<0.5
         % Teacher-Phase
         xNew=xMat(i,:)+rand*(Gbest-round(1+rand)*mean(xMat));
         GreedyReplacement;
      else
         % Observer-Phase
         for j=1:nVar
            xNew(j)=xMat(randi(PopSize),j);
         end
         GreedyReplacement;
      end
      % Learner-Phase
      tmp=randperm(PopSize-1); j=i+tmp(1); if j>PopSize,  j=j-PopSize; end
      if Fitness(j)>Fitness(i) % Fitness = -cost
         vNew=rand*(xMat(j,:)-xMat(i,:));
      else
         vNew=-rand*(xMat(j,:)-xMat(i,:));
      end
      xNew=xMat(i,:)+vNew;
      GreedyReplacement;
   end
   ElitistUpdate;
end
```

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