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SHAPE OPTIMIZATION OF STRUCTURES BY MODIFIED HARMONY SEARCH

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ABSTRACT

The main aim of the present study is to propose a modified harmony search (MHS) algorithm for size and shape optimization of structures. The standard harmony search (HS) algorithm is conceptualized using the musical process of searching for a perfect state of the harmony. It uses a stochastic random search instead of a gradient search. The proposed MHS algorithm is designed based on elitism. In fact the MHS is a multi-staged version of the HS and in each stage a new harmony memory is created using the information of the previous stages. Numerical results reveal that the proposed algorithm is a powerful optimization technique with improved exploitation characteristics compared with the standard HS.

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KEY WORDS: Shape optimization, harmony search algorithm, penalty functions, truss structure

1. INTRODUCTION

Structural optimization is a critical activity that has received considerable attention in the last four decades. Usually, structural optimization problems involve searching for the minimum of the structural weight. This minimum weight design is subjected to various constraints on performance measures, such as stresses and displacements, and also restricted by practical minimum cross-sectional areas or dimensions of the structural members or components. Due to considering these constraints the possibility of trapping in the local optima will be larger.

Optimum shape design of structures is one of the challenging research areas of the

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structural optimization field. In this class of optimization problems two types of design variables with different natures, including sizing and geometric variables, are involved. The shape optimization problem has been identified as a more difficult but more important task than pure sizing optimization, since potential savings in material can be far better improved than by the latter.

Most of the engineering optimization algorithms are based on numerical linear and nonlinear programming methods that require substantial gradient information and usually seek to improve the solution in the neighborhood of a starting point. These algorithms, however, reveal a limited approach to complicated real-world optimization problems. If there is more than one local optimum in the problem, the result may depend on the selection of an initial point, and the obtained optimal solution may not necessarily be the global optimum.

In the last years, structural optimization has been studied by using different natural phenomena based meta-heuristic algorithms. The most extensively applied meta-heuristic algorithms are Genetic Algorithm (GA) [1] Ant Colony Optimization (ACO) [2], Particle Swarm Optimization (PSO) [3] and etc. In the field of structural engineering many successful applications of the above mentioned algorithms have been reported in the literature [4-9]. The popularity of these algorithms is due to this fact that for implementation of the meta-heuristics the gradient of objective function and constraints are not required. In other words, they use a stochastic random search strategy instead of a gradient search so that derivative information is unnecessary. Also they are able to handle both discrete and continuous design variables and their computer implementation is simple.

One of the recent additions to meta-heuristics is the Harmony Search (HS) [10] method. The so called HS algorithm was recently developed by Lee and Geem in an analogy with music improvisation process where music player improvise the pitches of their instruments to obtain better harmony. Solution vectors in HS algorithm, is simulated with harmony in the music and search plan with artist initiative. In comparison with other meta-heuristics, HS imposes fewer mathematical requirements. These features increase flexibility of the HS to analysis various engineering optimization problems [11].

In this paper, a new multi-stage HS algorithm is proposed for size and shape optimization of structures. Also the exterior penalty function method (EPFM), due to its simplicity and ease of implementation, is employed in the framework of the sequential unconstrained minimization technique (SUMT) [12] to handle the constraints. The proposed algorithm is denoted as modified harmony search (MHS) algorithm.

Two benchmark structural shape optimization problems are solved by the proposed MHS. The numerical results indicate that the computational performance of the proposed MHS is better than that of the HS.

2. FORMULATION OF THE OPTIMIZATION PROBLEM

The mathematical formulation of structural optimization problems toward the design variables, the objective and constraint functions depend on the type of the application. However, all optimization problems can be expressed in standard mathematical terms, which in general form can be stated as follows:

$$\begin{aligned} &\text{Minimize} \quad F(X) \\ &\text{Subject to} \quad g_i(X) \le 0 \ , \quad i = 1, \dots, m \\ &X_j^l \le X_j \le X_j^u \ , \quad j = 1, \dots, n \end{aligned} \tag{1}$$

where, X is the vector of design variables; $F(X)$ is the objective function to be minimized; $g_i(X)$ is the ith behavioral constraints; X_j^l and X_j^u are the lower and the upper bounds on a typical design variable X_j.

In this study, to transform the constrained structural optimization problem into an unconstrained one the EPFM is employed. Penalty function methods transform the basic optimization problem into alternative formulations such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. The above mentioned constrained optimization problem can be converted into an unconstrained problem by constructing a function of the following form:

$$\Phi(X, r_p) = f(X) + r_p \sum_{i=1}^{m} [\max\{0, g_i(X)\}]^2 \tag{2}$$

where Φ, and r_p are the pseudo objective function, and positive penalty parameter, respectively.

By choosing the minor values for the penalty parameter, the effect of constraints in pseudo objective function decrease and optimization processes cause to minimize objective function with small amount of violated constraints, in the other side by choosing the high value for penalty parameter, the effect of constraints in pseudo objective function increases and the portion of objective function decreases. Vanderplaats recommended that if the unconstrained minimization of the pseudo objective function is repeated for a sequence of values of the penalty parameter, r_p, the solution may be brought to converge to that of the original problem. These methods are known as sequential unconstrained minimization techniques (SUMT). In the present study, the EPFM is employed in the framework of the SUMT to handle the constraints.

3. MODIFIED HARMONY SEARCH ALGORITHM

Harmony is defined as an attractive sound made by two or more notes being played at the same time. The new HS meta-heuristic algorithm was derived by adopting the idea that existing meta-heuristic algorithms are found in the paradigm of natural phenomena. The HS algorithm parameters that are required to solve the optimization problem are also specified in this step: harmony memory size (number of solution vectors, labled as HMS), harmony memory considering rate (HMCR), pitch adjusting rate (PAR), an arbitrary distance bandwidth for continuous variable (bw) and termination criterion (maximum number of searches). The basic steps of the HS may be mentioned as follows:

Step 1. Initialize algorithm parameters:

Specification of each design variable, a possible value range in each design variable, HMS, HMCR, PAR, and termination criterion are initialized.
Step 2. Initialize harmony memory:

The harmony memory (HM) matrix is filled with randomly generated designs as the size of the harmony memory (HMS).

Step 3. Improvise a new harmony from the HM:

New harmony vectors improvised from either the initially generated HM or the entire possible range of values. The new harmony improvisation progresses based on memory considerations, pitch adjustments, and randomization. Here, it is possible to choose the new value using the HMCR parameter, which varies between 0 and 1 as follows:

$$x_i^{'}\begin{cases} x_i^{'} \in \{x_i^1, x_i^2, \ldots, x_i^{HMS}\} & \text{w.p. of } HMCR \\ x_i^{'} \in X_i & \text{w.p. of } (1-HMCR) \end{cases} \tag{3}$$

where the *HMCR* is the probability of choosing one value from the significant values stored in the HM, and (1-*HMCR*) is the probability of randomly choosing one practical value not limited to those stored in the HM.

Every component of the new harmony vector is examined to determine whether it should be pitch-adjusted. This procedure uses the *PAR* parameters as follows.

$$\begin{cases} Yes & \text{w.p. of } PAR \\ No & \text{w.p. of } (1-PAR) \end{cases} \tag{4}$$

The pitch adjusting process is performed only after a value is chosen from the *HM*. The value (1-*PAR*) sets the rate of doing nothing. A *PAR* of 0.05 indicates that the algorithm will choose a neighboring value with 5% *HMCR* probability.

If the pitch adjustment applied for a design variable then a neighboring value with the probability of PAR%×HMCR is taken for it as follows:

$$\begin{aligned} x_i^{'} &\longleftarrow x_i(k+m) \\ x_i^{'} &\longleftarrow x_i + \alpha \end{aligned} \tag{5}$$

in which *m* is the neighboring index; α is the value of bw$\times u$(-1,1); bw is an arbitrary distance bandwidth for the continuous variable and u(-1,+1) is a uniform distribution between -1 and +1.

Step 4. Update the Harmony Memory:

If the new harmony is better than the worst vector in the HM the new solution vector is included in the HM and the existing worst vector is excluded from the HM.

Step 5. Termination Criteria:

Steps 3 and 4 are repeated until the termination criterion is satisfied. The termination criterion stops the algorithm when the maximum number of searches is reached.

In order to improve the exploration ability of the standard HS, the algorithm is employed in the framework of SUMT to solve the optimization problem and the design constraints are handled by EPFM. At first by choosing a minor r_p a HM with the size of HMS is initialized and the HS is employed to achieve a preliminary optimization task. The found solution in this manner may be infeasible. In the next step, the harmony vectors of new HM are selected from a neighboring region of the best solution obtained in the previous process. In this case, the best solution is directly transformed to the new HM and the remaining vectors are selected as the random numbers normally distributed about the mentioned best solution with the standard deviation of 10% times the best solution. After initializing a new HM, another optimization process is achieved by HS using an increased r_p. This procedure is repeated until the algorithm converges.

4. NUMERICAL EXAMPLE

In order to investigate the computational performance of the proposed MHS algorithm, two examples are presented. For all examples the HS parameters are as: HMS=10, HMCR=0.971, PAR = 0.05 and *bw* = 0.3. Also the maximum numbers of iterations in each optimization process and the maximum number of optimization processes are limited to 100 and 5, respectively (5000 structural analyses). All of the required computer programs are coded in MATLAB [13] platform.

4.1. 15-bar Truss

This problem has been investigated by Wu and Chow [14], Hwang and He [15], Tang et al. [16] and Rahami et al. [17]. The fifteen-bar 2-D truss is shown in Figure 1. the magnitude of the vertical load is P = 10 kips. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi.

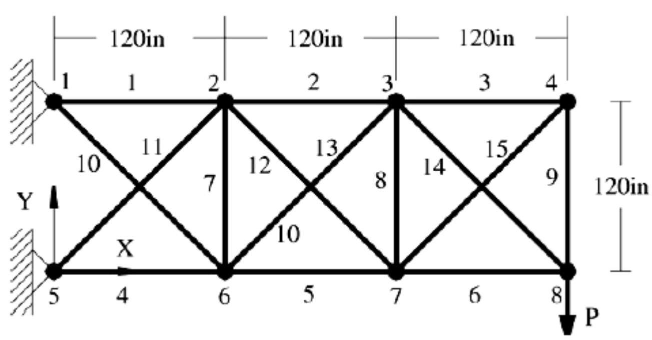


Figure 1. Fifteen-bar truss

In this example there are 23 design variables including two categories: Sizing variables: A_i, i=1,2,…,15 and Geometry variables: $x_2 = x_6$; $x_3 = x_7$; y_2; y_3; y_4; y_6; y_7; y_8. Stress limitation for all elements is $\pm 25\,\text{ksi}$.

The size variables are selected from the following set:

D = { 0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180} (in.^{2}). Also side constraints for geometry variables are as follows:

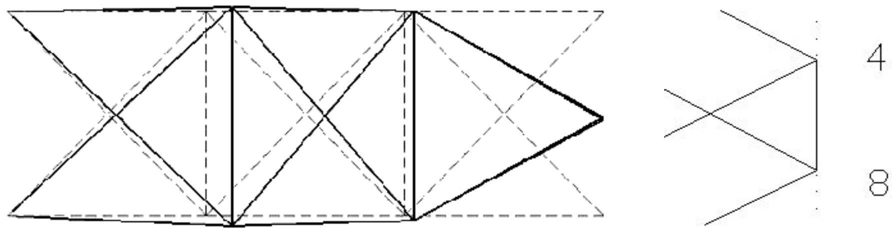
100 in. $\leq x_2 \leq$ 140 in.; 220 in. $\leq x_3 \leq$ 260 in.; 100 in. $\leq y_2 \leq$ 140 in.; 100 in. $\leq y_3 \leq$ 140 in.; 50 in. $\leq y_4 \leq$ 90 in.; 20 in. $\leq y_6 \leq$ 20 in.; −20 in. $\leq y_7 \leq$ 20 in.; 20 in. $\leq y_8 \leq$ 60 in.;

The best results obtained in this study are compared with those of the other works in Table 1.

Table 1. Optimal design comparison for the 15-bar planner truss

Design variables	Wu and Chow [14]	Hwang and He [15]	Tang et al. [16]	Rahami et al. [17]	Present work	
					HS	MHS
$A1$	1.174	0.954	1.081	1.081	1.081	0.954
$A2$	0.954	1.081	0.539	0.539	0.954	0.539
$A3$	0.440	0.440	0.287	0.287	0.270	0.220
$A4$	1.333	1.174	0.954	0.954	0.954	0.954
$A5$	0.954	1.488	0.954	0.539	0.539	0.539
$A6$	0.174	0.270	0.220	0.141	0.270	0.220
$A7$	0.440	0.270	0.111	0.111	0.111	0.111
$A8$	0.440	0.347	0.111	0.111	0.141	0.111
$A9$	1.081	0.220	0.287	0.539	0.220	0.440
$A10$	1.333	0.440	0.220	0.440	0.220	0.347
$A11$	0.174	0.220	0.440	0.539	0.440	0.347
$A12$	0.174	0.440	0.440	0.270	0.111	0.270
$A13$	0.347	0.347	0.111	0.220	0.440	0.270
$A14$	0.347	0.270	0.220	0.141	0.287	0.220
$A15$	0.440	0.220	0.347	0.287	0.220	0.220
$x2$	123.189	118.346	133.612	101.5775	137.2764	135.5676
$x3$	231.595	225.209	234.752	227.9112	220.0000	245.5421
$y2$	107.189	119.046	100.449	134.7986	138.5269	123.1303
$y3$	119.175	105.086	104.738	128.2206	127.4160	120.6957
$y4$	60.462	63.375	73.762	54.8630	50.0000	57.9313
$y6$	16.728	−20.0	−10.067	−16.4484	19.1800	−5.9742
$y7$	15.565	−20.0	−1.339	−16.4484	2.8000	−2.9125
$y8$	36.645	57.722	50.402	54.8572	38.3190	56.3256
Best weight (lb)	120.52	104.573	79.820	76.6854	80.364	73.887
Number of analyses	-	-	8000	8000	5000	5000

The results demonstrate the computational advantages of the MHS with respect to other algorithms. The geometry of the optimum structure is shown in Figure 2.



(a) (b)

Figure 2. (a) Optimum shape of fifteen-bar planar truss (b) geometry of nodes 4 and 8

To assess the computational performance of the proposed MHS algorithm, 25 independent runs are achieved and the best, worst and mean weights of 73.887 lb, 88.420 lb and 79.206 lb are obtained.

4.2. 25-bar truss

This problem has been investigated by Wu and Chow [14], Tang et al. [16] and Rahami et al. [17]. The twenty five-bar truss is considered as shown in Figure 3.

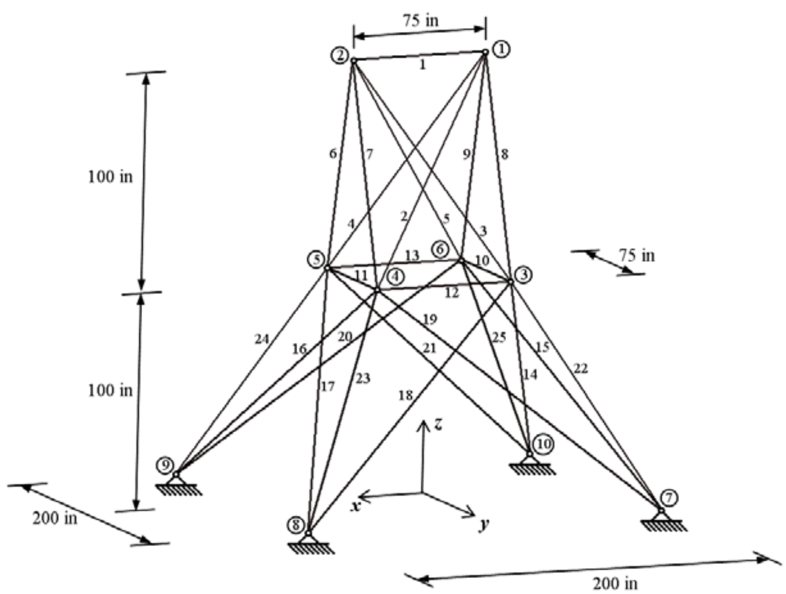


Figure 3. Twenty five-bar space truss

The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi. Loading data is given in Table 2.

Table 2. Loading data for twenty five-bar truss

Node	F_x (kips)	F_y (kips)	F_z (kips)
1	1.0	-10.0	-10.0
2	0.0	-10.0	-10.0
3	0.5	0.0	0.0
6	0.6	0.0	0.0

There are 13 design variables including two categories as follows:

Size variables: A_1; $A_2 = A_3 = A_4 = A_5$; $A_6 = A_7 = A_8 = A_9$; $A_{10} = A_{11}$; $A_{12} = A_{13}$; $A_{14} = A_{15} = A_{16} = A_{17}$; $A_{18} = A_{19} = A_{20} = A_{21}$; $A_{22} = A_{23} = A_{24} = A_{25}$

Geometry variables: $x_4 = x_5 = -x_3 = -x_6$; $x_8 = x_9 = -x_7 = -x_{10}$; $y_3 = y_4 = -y_5 = -y_6$; $y_7 = y_8 = -y_9 = -y_{10}$; $z_3 = z_4 = z_5 = z_6$

Stress limitation for all elements is $\pm 40\,\mathrm{ksi}$ also displacement constraint in all directions is 0.35 in. The size variables are selected from the following set:

D = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4} (in.^{2}).

Also side constraints for geometry variables are as follows:

20 in. $\leq x_4 \leq$ 60 in.; 40 in. $\leq x_8 \leq$ 80 in.; 40 in. $\leq y_4 \leq$ 80 in.; 100 in. $\leq y_8 \leq$ 140 in.;

90 in. $\leq z_4 \leq$ 130 in.;

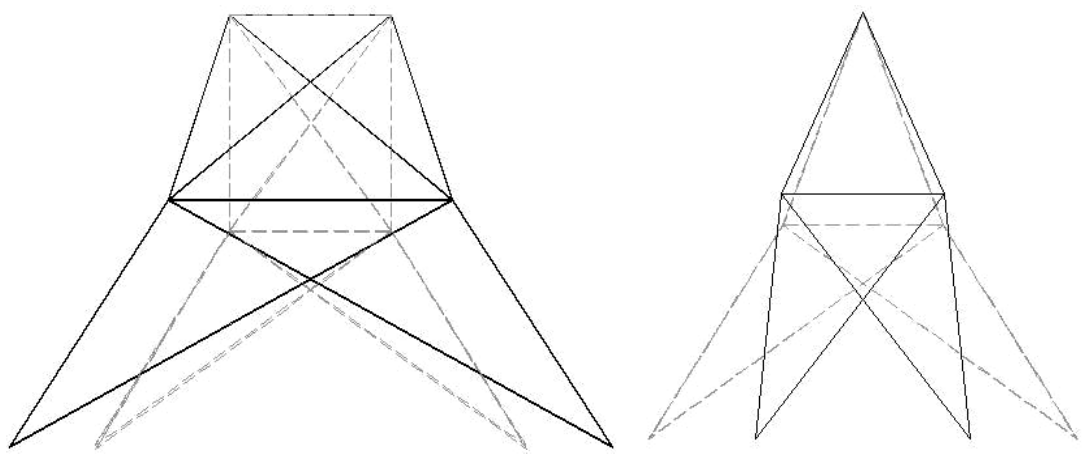
The best results obtained in this study are compared with those of the others in Table 3.

Table 3. Optimal design comparison for the 25-bar space truss

Design variables	Wu and Chow [14]	Tang et al. [16]	Rahami et al. [17]	Present work	
				HS	MHS
A1	0.1	0.1	0.1	0.1	0.1
A2	0.2	0.1	0.1	0.1	0.1
A3	0.1	1.1	1.1	1.0	0.1
A4	0.2	0.1	0.1	0.1	0.1
A5	0.3	0.1	0.1	0.1	0.1
A6	0.1	0.2	0.1	0.1	0.1
A7	0.2	0.2	0.2	0.1	0.1
A8	0.9	0.7	0.8	1.0	0.9
X4	41.07	35.47	33.0487	32.95	37.82
Y4	53.47	60.37	53.5663	68.185	55.485
Z4	124.6	129.07	129.9092	107.37	128.73
X8	50.8	45.06	43.7826	47.367	52.068
Y8	131.48	137.04	136.8381	136.02	139.59
Best weight (lb)	136.20	124.94	120.11	122.62	117.38
Number of analyses	-	6000	8000	5000	5000

The results demonstrate the computational advantages of the MHS with respect to other algorithms. The geometry of the optimum structure is shown in Figure 4.

To assess the computational performance of the proposed MHS algorithm, 25 independent runs are achieved and the best weight of 117.40 lb, the worst weight if 130.20 lb and the mean weight of 119.02 lb are obtained.



(a) (b)

Figure 4. (a) Optimum shape of truss in (a) y-z plan and (b) x-z plan

5. CONCLUSION

The shape optimization of truss structures is tackled in this paper using an enhanced HS meta-heuristic algorithm. In order to improve the computational performance of HS the standard HS algorithm is sequentially utilized in the framework of SUMT employing EPFM to handle the design constraints. The proposed meta-heuristic algorithm is termed as modified harmony search (MHS) algorithm. Both size and shape structural optimization problems are solved by the proposed algorithm and the results are compared to those of the other researchers. The numerical results indicate that using MHS not only better solutions can be found but also a significant reduction in computational effort may be achieved. For more details, in the case of first example, the best weight found in the literature is 76.6854 lb spending 8000 structural analyses while in the present paper the best weight of 73.887 lb is obtained after 5000 structural analyses. In the case of second example, the weight of the best structure and its corresponding required number of analyses reported in the literature are 120.11 lb and 8000, respectively while these values in the present paper are 117.38 lb and 5000, respectively. These results emphasize on the efficiency of the proposed MHS algorithm for shape optimization of structures.

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