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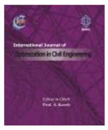

ANFIS–TLBO HYBRID APPROACH TO PREDICT COMPRESSIVE STRENGTH OF RECTANGULAR FRP COLUMNS

M. R. Ghasemi[*, †], M. Ghasri and A.H. Salarnia
Department of Civil Engineering, University of Sistan and Baluchestan, Zahedan, Iran

ABSTRACT

Today, due to the complexity of engineering problems and at the same time the advancement of computer science, the use of machine learning (ML) methods and soft computing methods in solving engineering problems has been considered by many researchers. These methods can be used to find accurate estimates for problems in various scientific fields. This paper investigates the effectiveness of the Adaptive Network-Based Fuzzy Inference System (ANFIS) hybridized with Teaching Learning Based Optimization Algorithm (TLBO), to predict the ultimate strength of columns with square and rectangular cross-sections, confide with various fiber-reinforced polymer (FRP) sheets. In previous studies by many researchers, several experiments have been conducted on concrete columns confined by FRP sheets. The results indicate that FRP sheets effectively increase the compressive strength of concrete columns. Comparing the results of ANFIS-TLBO with the experimental findings, which were agreeably consistent, demonstrated the ability of ANFIS-TLBO to estimate the compressive strength of concrete confined by FRP. Also, the comparison of RMSE, SD, and R^2 for ANFIS-TLBO and the studies of different researchers show that the ANFIS-TLBO approach has a good performance in estimating compressive strength. For example, the value of R^2 in the proposed method was 0.92, while this parameter was 0.87 at best among the previous studies. Also, the obtained error in the prediction of the proposed model is much lower than the obtained error in the previous studies. Hence, the proposed model is more efficient and works better than other techniques.

Keywords: ANFIS; Teaching Learning Based Optimization; FRP; compressive strength.

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[*]Corresponding author: Department of Civil Engineering, University of Sistan and Baluchestan, Zahedan, Iran
[†]E-mail address: mrghasemi@eng.usb.ac.ir (M.R. Ghasemi)

1. INTRODUCTION

The advancement of science in computer science and its combined use with other scientific disciplines has caused analysis of many complex problems to be done more inexpensively and even in less time [1, 2]. The use of soft computing methods in solving complex engineering problems has become one of the most important topics in recent years. Soft computing refers to a calculation based on artificial intelligence. The term soft computing was coined by Prof. Zadeh [3]. The conception of soft computing in the computing world was characterized by research in machine learning, probabilistic logic, artificial neural networks (ANN), fuzzy logic [4]and genetic algorithm (GA)[5, 6]. In principle, soft computing refers to selecting cost-effective and acceptable solutions to complex computational problems. This method provides approximately acceptable solutions to problems that existing hardware cannot solve or require lots of time. By accepting the human mind as a role model, the soft computing approach can, unlike traditional computational models, accept actualities such as uncertainty, inaccuracy, and approximation in solving complex problems[7, 8].

This feature of soft computing authorizes researchers to analyze some of the problems that traditional computing cannot handle. As mentioned, the combined use of soft computing and other sciences can increase the ability to solve challenging and time-consuming problems. One of the crucial matters in civil engineering that has always been discussed and studied is the construction of strong structures and, more importantly, retrofitting existing structures. Due to the fact that it has a direct relationship with human health and life, today there is a need for new methods to build and strengthen structures [9-11].

With the advancement of computer science and its integration with civil engineering, new methods can be used to retrofit structures. Before starting the discussion, it should be noted that the retrofitting of existing structures is debatable and valuable because the structures during construction or even after construction need retrofitting for various reasons. Examples of these errors include the following:

1. errors in the construction phase
2. errors in the design phase
3. corrosion of building materials (such as reinforcements)
4. changes in design regulations, etc.

One of the main subjects in retrofitting structures is to predict the behavior of the member or part of the structure on which the retrofit operation has been performed. Predicting the behavior of a structure after retrofitting can greatly help engineers improve the design process. In this paper, the reinforcement of reinforced concrete columns using fiber-reinforced polymer composite sheets is specifically investigated. In recent years, FRP has been used as a practical method for reinforcing reinforced concrete columns. Its advantages include short production and installation time, lightweight, long-term cost savings, corrosion resistance, and long life [12]. One common way to work with FRP to strengthen columns is to wrap them around a concrete column to increase its axial strength and ductility. It is well known that a concrete core expands laterally when subjected to uniaxial pressure, but FRP sheets inhibit this expansion. Thus, the core is subjected to a three-dimensional compressive state of stress in which the performance of the concrete core is significantly affected by the limiting pressure [11, 13, 14].

Predicting and estimating the compressive strength of reinforced concrete columns after reinforcement with FRP sheets has been one of the topics of interest in the combined application of soft computation and civil engineering methods[15]. The most widely used soft computing methods developed and used in this field include machine learning, artificial neural networks, optimization algorithms and fuzzy logic[16].

The application of soft computing methods is such that first a database of independent or dependent variables and parameters for estimating compressive strength in laboratories is generated experimentally and then this database is given to a computer. Then using the mentioned methods, a model of mathematical solutions between dependent and independent variables is presented. In fact, using soft computing methods, a reliable model for predicting compressive strength using experimental databases is presented[17, 18].

In the meantime, using optimization algorithms, we try to optimize the parameters of the proposed model as much as possible to increase the efficiency of the model. In this paper, the authors examine the performance of a hybrid soft computing method that includes an Adaptive Network-Based Fuzzy Inference System (ANFIS) and Teaching Learning Based Optimization (TLBO) algorithm[19] to predict the compressive strength of square and rectangular FRP confined columns.

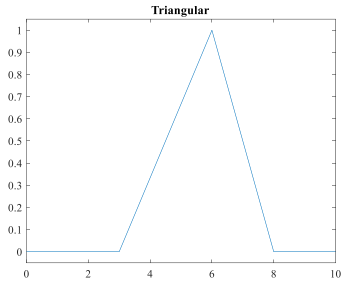
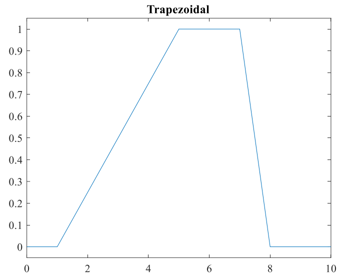
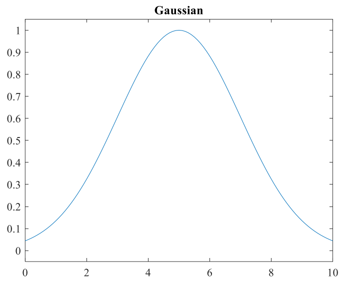
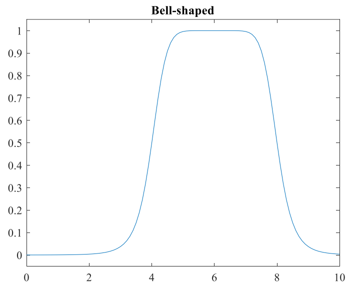
2. ADAPTIVE NETWORK-BASED FUZZY INFERENCE SYSTEM (ANFIS)

ANFIS is an intelligent system combining neural network theories and fuzzy control in which the advantages of the two methods are integrated. This method was introduced in 1993 by Jang [4]. In this system, which consists of several layers, nodes in different layers take fuzzy parameters forward. This is analogous to fuzzy inference systems (FIS) with distributed parameters. At its gut, the technique divides prior knowledge into subcategories to reduce search space and uses a backpropagation algorithm to modify fuzzy parameters. The resulting system is an adaptive neural network that is functionally equivalent to a first-order Takagi-Sugeno inference system, in which the input-output relationship is linear[20]. According to Fig. 1, the concept of ANFIS structure includes five separate layers; these five layers are:

1. Layer 1 – Fuzzification

In this layer, the model classifies inputs based on fuzzy membership functions (MFs) with adjustable parameters. The MFs can abide under any shape or function, such as triangular, trapezoidal, gaussian, or bell-shaped. Table 1 shows the mathematical equation of the functions and their diagrams. The membership functions considered in this study are Gaussian shaped.

Table 1: Popular membership functions

MFs	Equation	Diagrams
Triangular	$f(x;a,b,c) = max\left(min\left(\frac{x-a}{b-a},\frac{c-x}{c-b}\right),0\right)$	
Trapezoidal	$f(x;a,b,c,d) = max\left(min\left(\frac{x-a}{b-a},1,\frac{d-x}{d-c}\right),0\right)$	
Gaussian	$f(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$	
Bell-shaped	$f(x;a,b,c) = \frac{1}{1+\left\lvert\frac{x-c}{a}\right\rvert^{2b}}$	

a, b, c and d are the parameter set that changes the shapes of the MFs with maximum 1 and minimum 0. σ is the standard deviation of the Gaussian function.

Construction of the ANFIS model requires splitting the input and output data into rule patches. This data division can be done using three methods: Grid Partitioning (GP), Subtractive Clustering method (SC) and Fuzzy c mean (FCM). The Method considered in this study is FCM [7].

1.1 FCM method

FCM is one of the most common methods in fuzzy logic clustering, introduced in 1973 by Joseph Dunn[21]. The function of the FCM algorithm is to divide the input data into groups or clusters required for fuzzy logic and specify the center of each cluster to perform operations on the data. James Bezdek later explored this algorithm in 1984[22], and an improv version was released. Since the details about the mathematical of ANFIS can be found in numerous literatures [4, 10, 11] they are not explained in this paper.

2. Layer 2 – Rules

This layer is illustrated with the Π symbol in Fig. 1. In this layer, the rule operator (AND/OR) is applied to get one output representing the antecedent results for a fuzzy rule that multiplies the incoming signals. Briefly, this layer deliberates the weight of MFs.

3. Layer 3 – Normalization

The normalization layer denotes the interpolation layer, and the nodes of this layer are shown in Fig. 2 with N. This layer represents the firing strength of the rule i-th to the sum of the firing strength of all rules.

4. Layer 4 – Defuzzification

A layer that executes the output, resulting from the inference of rules and multiplies them with the Sugeno fuzzy rule’s function.

5. Layer 3 – Cumulative

That comprises the weighted average summation method to calculate the network output.

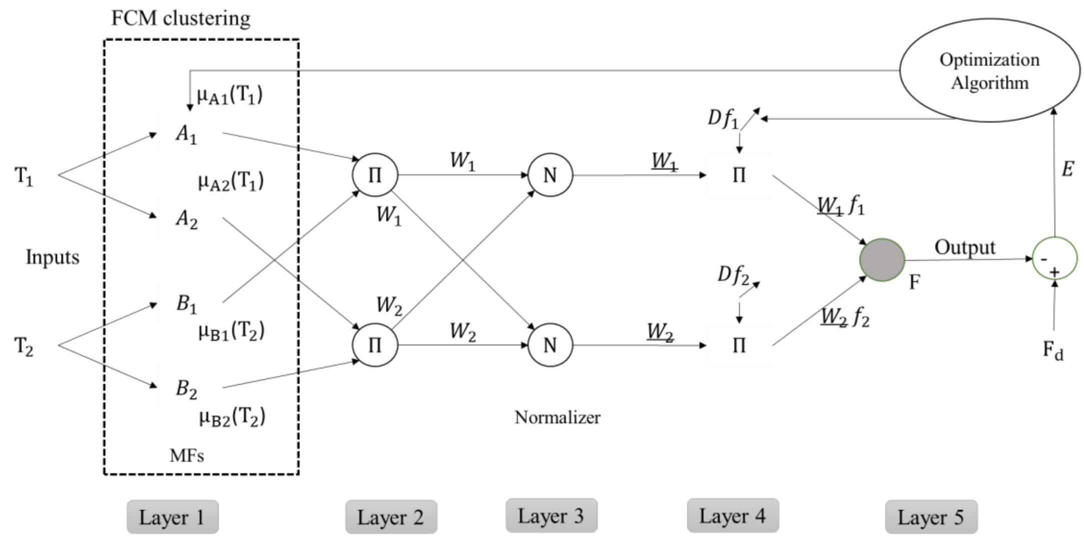


Figure 1. Basic structure of ANFIS

3. TEACHING LEARNING BASED OPTIMIZATION (TLBO)

To date, various studies on the TLBO algorithm have been performed by researchers, and the results obtained from these studies show the high reliability of this algorithm in obtaining the final optimal answer to various optimization problems. These references can be mentioned from the studies done on the TLBO algorithm. This algorithm solves problems by simulating a classroom. The TLBO algorithm has two main steps: the Teaching phase and the Learner (student) phase.

In the teaching phase where the learner with the highest marks (fitness function value) conducts as a teacher, and the teacher must improve the mean marks of the class. The update process of i-th learner in the teacher phase is formulated as:

$$\begin{aligned} &Var_{i,new} = Var_i + R \times (Var_{\text{teacher}} - T_F \times Var_{ave}) \\ &T_F = round \times [1 + rand(0,1)\{2-1\}] \end{aligned} \tag{1}$$

where, Var_i is the solution of the i-th learner, Var_{teacher} represents the teacher's solution, Var_{ave} indicates the average of all learners. R is a random number in (0,1), and T_F is the teaching factor that decides the mean value to be changed. The value can be either 1 or 2, a heuristic step that is selected randomly with equal probability T_F. The new solution $Var_{i,new}$ is accepted when its fitness function value ($fit(Var_{i,new})$) is improved than the previous value (Briefly, applying greedy selection).

The Learner phase is where the student updates their knowledge through interaction with other students. In each iteration, two students interact with Var_m and Var_n, in which the more innovative student improves the marks of other students. In the learner phase, one student learns new things if the other learner has more knowledge than himself. The mathematical form is described as follows: (In this phase greedy selection is also applied.)

$$Var_{m,new} = \begin{cases} Var_m + R \times (Var_m - Var_n) & \rightarrow \quad fit(Var_m) > fit(Var_n) \\ Var_m + R \times (Var_n - Var_m) & \rightarrow \quad fit(Var_n) > fit(Var_m) \end{cases} \tag{2}$$

The algorithm has two parameters: the number of students (Num_S) and the maximum number of fitness function evaluations (Max_E). Since these two parameters exist in any other meta-heuristics, TLBO can be called a parameter-less meta-heuristic. The pseudo-code of TLBO illustrate as follows (Table 2):

Table 2: The pseudo-code of TLBO algorithm.

// Initialization:

- Set the algorithm parameters: Num_S and Max_E;
- Generate initial students (S) in a random manner;
- Evaluate the objective function (i.e. the knowledge of initial students) of all the candidate solutions;

// The main Loop:

While iter ≤ Max_E

Step 1: Generate new students ($Var_{i,new}$) based on the teacher phase (Eq.1);
Step 2: Evaluate the $Var_{i,new}$ and applying greedy selection;
Update Fitness functions values.

Step 3: Generate new students ($Var_{m,new}$) based on the learner phase (Eq.2);
Step 4: Evaluate the $Var_{m,new}$ and applying greedy selection;
Step 5: Monitor the best student;
End While

4. METHODOLOGY

The idea here is to employ the TLBO algorithm to optimize ANFIS parameters to improve performance. ANFIS provides the search space and utilizes TLBO for finding the best solution by tuning the membership functions required to achieve lower error rates. The error between the model output and the exact training data can reach a minimum value by iterating the TLBO algorithm until the desired error is met. The ANFIS– TLBO approach was coded with the MATLAB software.

5. EXPERIMENTAL DATABASE

In recent years, many studies have been carried out on concrete confined by FRP sheets. In previous studies, various machine learning methods have been used to estimate the compressive strength of rectangular columns confined by FRP plates. Table 3 reports these studies, along with the number of samples, the machine learning method. As shown in Table 3, quantitative methods have been used to estimate the compressive strength of rectangular columns so far. In this research work, a number of 463 concrete samples were used to proceed. They were selected from a database collected by reference as follows: Moodi et al. [23].

The existing square and rectangular specimens of this statistical population include the width (b) of 70:450 mm, the length (h) of 70:600 mm, the corner radius (r) offset of 0:60 (mm), and the unconfined compressive strength (f_{co}) of 10:110.8 MPa. Also, different types of FRPs, including CFRP, AFRP, and GFRP, exist in this database. A random 70% of these specimens were used as training samples and 30% for evaluation and testing procedure.

Table 3: Methods used to estimate compressive strength of rectangular FRP-confined columns

Study	Year	Methods	Types of concrete	No. specimens
Wu et al. [24]	2010	RBFNN	Plain	154
Pham and Hadi [25]	2014	ANN	Plain	209
Doran et al [26]	2015	MFIS	Plain	140
Moodi et al [27]	2018	RSM	Plain	416
Sharifi et al [28]	2019	ANN	Plain	190
Mohana [29]	2019	ANN, SVR	RC	163

Moodi et al [23]	2021	MLP, RBFN, SVR	Plain	463

6. RESULTS AND DISCUSSIONS

In this section, the performance of ANFIS- TLBO for estimating the compressive strength of square and rectangles columns bounded by FRP is compared and discussed. Frequently used indicators, including Standard Deviation (SD) and Root Mean Square Error (RMSE) were used for this evaluation. Equations SD and RMSE are defined as follows:

$$SD = \sqrt{\frac{\sum_1^N \left(\frac{Th_i}{Ex_i} - \frac{Th_{avg}}{Ex_{avg}}\right)^2}{N-1}} \tag{3}$$

$$\text{RMSE} = \sqrt{\frac{\sum_1^N \left(\frac{Th_i - Ex_i}{Ex_i}\right)^2}{N}} \tag{4}$$

where N is the number of samples and Ex_i and Th_i are the experimental and ANFIS-based theoretical compressive strengths, respectively. Training and testing results for predicting the strengths of the ANFIS- TLBO model are listed in Table 4. Fig. 2. illustrates the strength estimates of the applied models in the forms of the scatterplot.

Table 4: Statistical performance ANFIS- TLBO

Method	Train phase			Test phase		
	RMSE	SD	R^2	RMSE	SD	R^2
ANFIS-TLBO	0.94	7.86	0.94	1.12	10.36	0.92

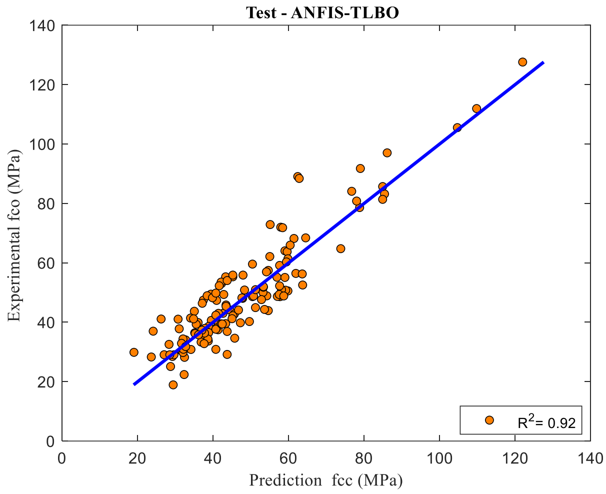


Figure 2. Performance of the model

In order to compare the result of the model more concretely, the amount of concrete compressive strength for all samples is also calculated with the equations listed in Table 4 (the equations are adapted from other existing articles). The results are shown in Table 5 with three statistical indices, correlation coefficient criteria (R^2), Root Mean squared error (RMSE), and Standard Deviation (SD) (Equations 3,4).

Table 4: The Models used for FRP-Confined Concrete Compressive Strength

Reference	Model	Description
Moodi et al. [17]	$f'_{cc} = f'_c(1+3.3k_\varepsilon \frac{f_{l,a}}{f'_c})$	$f_{l,a} = \frac{2E_{frp}t_j\varepsilon_{frp}}{D}$ $k_\varepsilon = \frac{\pi r + 0.1996b + 0.0107h}{b+h-(4-\pi)r}, \quad D=\sqrt{b^2+h^2}$
Wei and Wu [30]	$f'_{cc} = f'_c(1+2.2(\frac{2r}{b})^{0.72}(\frac{f_{l,a}}{f'_c})^{0.72}(\frac{h}{b})^{-1.9}$	$f_{l,a} = \frac{2F_{frp}t_j}{b}$
Lam and Teng [31]	$f'_{cc} = f'_{co}(1+3.3k_a \frac{f_{l,a}}{f'_{co}})$	$f_{l,a} = \frac{2E_{frp}t_j\varepsilon_{h,rup}}{D}$ $\varepsilon_{h,rup} = k_\varepsilon \varepsilon_{frp}, \quad D=\sqrt{b^2+h^2}$ $k_a = (\frac{b}{h})^2[1-\frac{(\frac{b}{h})(h-2r)^2+(\frac{h}{b})(b-2r)^2}{3A_g}]$
Toutanji et al. [32]	$f'_{cc} = f'_c + 4(\frac{2r}{D})^{0.1}(\frac{h}{b})^{0.13}k_a f_{l,a}$	$f_{l,a} = \frac{2E_{frp}\varepsilon_{fe}t_j}{D}, \; D=\frac{2bh}{h+b}$ $k_a = 1-\frac{(b-2r)^2+(h-2r)^2}{3bh}$

Table 5: Statistical performance of ANFIS- TLBO

Method	Index		
	RMSE	SD	R^2
ANFIS- TLBO	1.23	11.26	0.91
Moodi et al. [17]	1.55	15.59	0.87
Wei and Wu [30]	1.93	18.74	0.86
Lam and Teng [31]	2.29	22.63	0.82
Toutanji et al. [32]	2.53	23.23	0.82

Needless to say, the ANFIS- TLBO correlation error was lower than the compared models and it was able to predict the compressive strength with appropriate accuracy.

7. CONCLUSION

The use of soft computing methods in the processing of engineering problems has made it possible to solve difficult and complex problems in a short time and easily with the help of this system. Predicting structural behavior is one of the most important applications of soft computing science. This paper discussed one of the combined soft calculation methods for predicting the compressive strength of reinforced concrete columns confined with FRP sheets. This method combines the Adaptive Network-Based Fuzzy Inference System (ANFIS) and Teaching Learning Based Optimization (TLBO) algorithm.

Fuzzy logic starts solving and processing problems by placing the human mind as the main role. ANFIS system consists of several sub-layers, each of which has weights and coefficients that can be found by finding the optimal value of these coefficients to increase the system's overall performance. To optimize these coefficients in this paper, the TLBO algorithm was used, which is one of the simple and robust algorithms for solving dimensional optimization problems.

R^2, RMSE and SD methods were used to evaluate the model. To train and test these methods, a comprehensive database containing 463 samples of FRP- confined rectangular/square concrete was used.

According to the results, the use of the TLBO algorithm in the ANFIS structure boosted the model compared to the previous studies. Also, it can be noted that the error obtained in the test phase is much lower than the methods of previous studies, for example, the value of R^2 in the proposed method was 0.91, while this parameter was 0.87 at best among the models. Also, the obtained error in the prediction of the proposed model is much lower than the obtained error in previous studies. Hence, the proposed model is more efficient and works better than other techniques.

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