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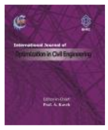

A MODIFIED SINE-COSINE ALGORITHM WITH DYNAMIC PERTURBATION FOR EFFECTIVE OPTIMIZATION OF ENGINEERING PROBLEMS

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ABSTRACT

The sine-cosine algorithm is concerned as a recent meta-heuristic method that takes benefit of orthogonal functions to scale its walking steps through the search space. The idea is utilized here in a different manner to develop a modified sine-cosine algorithm (MSCA). It is based on the controlled perturbation about current solutions by applying a novel combination of sine and cosine functions. The desired transition from exploration to exploitation phases mainly relies on such a term that provides continued fluctuations within a dynamic amplitude. Performance of the proposed algorithm is further evaluated on a set of thirteen test functions with unimodal and multimodal search spaces, as well as on engineering and structural problems in a variety of discrete, continuous and mixed discrete-continuous types. Numerical simulations show that MSCA can find the best literature results for such benchmarks problems. Additional fair comparisons, declare competitive performance of the proposed method with other meta-heuristic algorithms and its enhancement with respect to the standard sine-cosine algorithm.

Keywords: Sine-cosine algorithm; computational intelligence; meta-heuristic method; function optimization; constrained problem.

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1. INTRODUCTION

Many meta-heuristic algorithms can search for the global optimum without calculation of derivatives. Hence, they are popular solutions for various fields of engineering and structural problems [1–3]. The Sine-Cosine Algorithm (SCA) is one of the recent meta-heuristic

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methods that utilizes orthogonal trigonometric (sine and cosine) functions for its search [4]. A number of improving techniques have already been offered for SCA to overcome challenges such as premature convergence, low effectiveness or efficiency. Sindhu et al. [5] introduced an Improved Sine Cosine Algorithm (ISCA) by applying an elitism strategy that preserves the updated best solution and enhances the classification accuracy. Suid et al. [6] presented a Modified Sine Cosine Algorithm (M-SCA) by updating the step size gain and also the design vector so that the original structure of SCA is preserved. Gupta and Deep [7] used the crossover operator and modified the searching steps of SCA by employing the global best or a social direction into its walks. This way, they added the information of the best-so-far position in the memory to the candidate solutions. A Multi-group Multi-strategy Sine Cosine Algorithm (MMSCA) was proposed by Yang et al. [8]; in which multiple populations are utilized in a parallel manner but each population employs a different optimization strategy. An improved sine cosine algorithm with was proposed by Gao et al. [9] so that the population members update their positions in three ways: a part are reinitialized, another part obey the traditional SCA walks, and the remained part of the population utilize their own historical trajectories. Xian et al. [10] proposed a hybrid teacher supervision learning with SCA (TSL-SCA). Embedding opposition-based learning, adaptive evolution and neighborhood search are other techniques introduced by Feng et al. [11], to improve the sine cosine algorithm. Yang et al. [12] developed a multi-mechanism acting variant (ARSCA) for more balanced state between exploration and exploitation, and for enhancing the local exploitation capabilities of SCA. In this regard, they utilized an Adaptive Quadratic Interpolation Mechanism (AQIM) and a Rounding Mechanism (RM). More recent surveys on the applications and variants of the sine-cosine algorithm can be found in literature [2,13].

The present work offers a new variant of the standard sine-cosine algorithm to efficiently seek for the solutions around the global optima. In this regard, a perturbation factor is utilized so that exploration and exploitation can be balanced by a tuned envelope function. Details of the proposed algorithm is give in Section 3 after a brief review of the standard sine-cosine algorithm in Section 2. Consequently, performance of the new algorithm is evaluated on unconstrained optimization by solving thirteen unimodal and multimodal test functions in Section 4. It is followed by Section 5 representing the numerical results of the constrained optimization for a number of benchmark engineering and structural problems in a variety of discrete, continuous and mixed discrete-continuous types. Fair comparisons are provided with the standard SCA [4] as well as eight other algorithms including Particle Swarm Optimization (PSO) [14], Stochastic Paint Optimizer (SPO) [15], Falcon Optimization Algorithm (FOA) [16], Multi-Verse Optimizer (MVO) [17], Lightning Attachment Procedure Optimization (LAPO) [18], Marine Predators Algorithm (MPA) [19], Bald Eagle Search (BES) [20] and Manta Ray Foraging Optimization (MRFO) [21]. The present study is concluded in Section 6.

2. THE STANDARD SINE COSINE ALGORITHM

Since the sine-cosine algorithm (SCA) was introduced by [4] as an innovative meta heuristic method, its variants have been addressed in various applications [2, 22]. The standard SCA is inspired by mathematical characteristics of the sine and the cosine functions. SCA utilizes the following formula to update the positions of its population of search agents:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^{t} + r_1 \times \sin(r_2) \times \left| r_3 X_{Gbest,j}^{t} - X_{i,j}^{t} \right|, & r_4 < 0.5 \\ X_{i,j}^{t} + r_1 \times \cos(r_2) \times \left| r_3 X_{Gbest,j}^{t} - X_{i,j}^{t} \right|, & r_4 \geq 0.5 \end{cases} \tag{1}$$

At any iteration t, the j^{th} component of the i^{th} agent is shown by $X_{i,j}^{t}$ while $X_{Gbest,j}^{t}$ denotes the corresponding component of the best-so-far point position. The sign | | stands for the absolute value. The random numbers: r_2, r_3 and r_4 are uniformly distributed within the interval [0,1]. The factor r_1 is utilized for transition between the exploration and exploitation phases by:

$$r_1 = a - a \times (\frac{t}{T}) \tag{2}$$

The current iteration number is t while its maximum is denoted by T. The parameter α is usually fixed to 2. Fig. 1a shows schematic flowchart of the sine cosine algorithm.

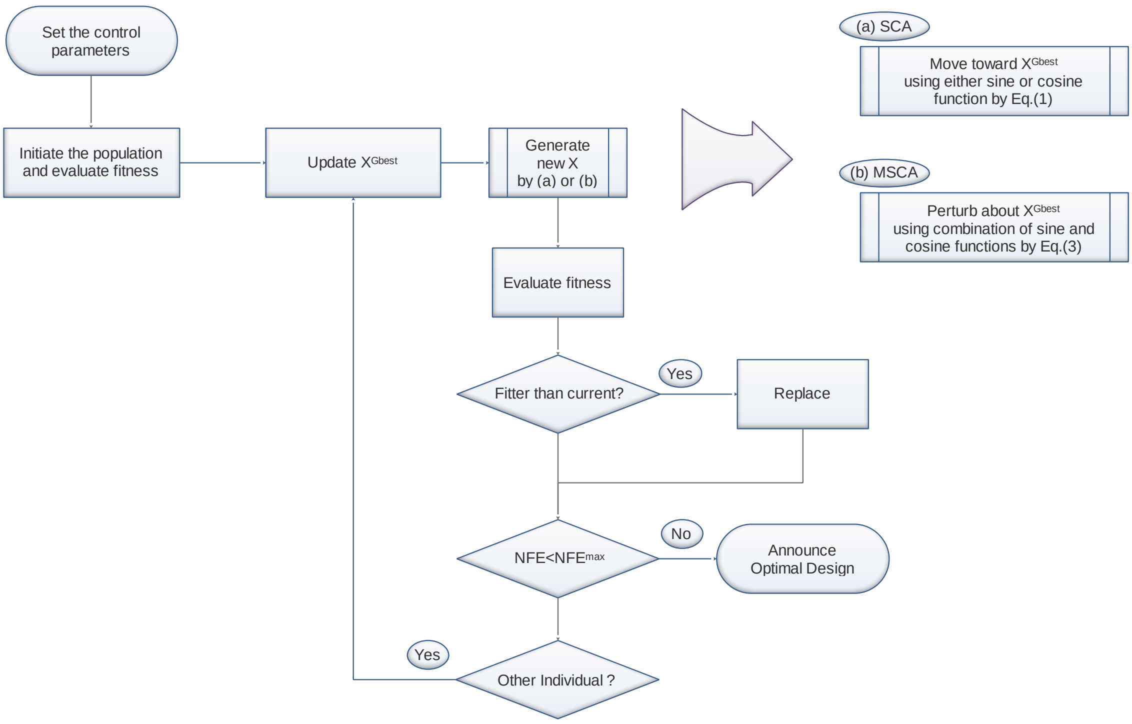

Figure 1. Flowchart of (a) Sine Cosine algorithm (b) Modified Sine Cosine algorithm

3. THE PROPOSED ALGORITHM

Every population-based algorithm employs several individual agents to parallel the search process in various regions of the design space. Consequently, the best position found up to the current iteration, can be used to guide the search toward true global optimum. Such a global best position at the iteration t is denoted by the vector: $X^t_{G_{best}}$. The proposed method, here-in-after called the Modified Sine-Cosine Algorithm (MSCA), utilizes a common strategy to share the information given by $X^t_{G_{best}}$ with the current position of each individual X^t. The strategy is implemented in MSCA by Eq. (3).

$$X_i^{t+1} = X_{G_{best}}^t + PF \times X_i^t \tag{3}$$

However to form the candidate solution X^{t+1} at every iteration, the priority given to the individual position is different from that given to the global best. It is utilized by introducing the following *Perturbation Factor* (PF):

$$PF = r \times \left[\cos\left(2\pi \times Rand\right) - \sin\left(2\pi \times Rand\right) \right] \tag{4}$$

The function $Rand$ stands for a random number generator between 0 and 1. The transition factor r is given by:

$$r = a \times \left[1 - \left(\frac{t - T \times d}{T - T \times d} \right)^c \right]^b \tag{5}$$

The parameters a, b, c and d are set prior to start the optimization process. Every meta-heuristic algorithm has its own method of balancing between exploration and exploitation. Exploration is the process of searching for new solutions while exploitation refers to the refinement about the currently found solutions. As it turns out, promoting one usually corresponds to decreasing the other. The proposed algorithm uses a different strategy with respect to the standard SCA to apply such a balance. According to Eq. (4) PF varies between $-2r$ to $2r$; as a scale of the current solution vector to be added to the global best. The factor r dynamically tunes such a domain as the search progresses from the beginning to the end. Fig. 1b shows the flowchart of the proposed MSCA; that is briefed via the following steps:

Step 1. Initiate N individual agents of the population by randomly positioning every i^{th} individual within the lower and the upper bounds on the design vector (X_L and X_U), respectively:

$$X_i = X_L + Rand \otimes (X_U - X_L) \tag{6}$$

Set the iteration number to 1 and fix the parameters a, b, c and d.

Step 2. Evaluate the cost function for all the individuals and identify their best as $X^t_{G_{best}}$.

Step 3. Repeat the following main loop until the termination criterion is satisfied:

- For t form 2 to T do
  - ◦ Calculate r by using the Eq. (5)
  - ◦ For i from 1 to the population size (N) do
    - • Update the position of individuals by using the Eq. (3)
    - • Force the new position to be within the lower and the upper bounds
    - • Evaluate the cost of X_i^{t+1}
    - • If the cost of X_i^{t+1} is lower than X_i^t, replace them by 50% chance otherwise keep them unchanged
    - • Exit the main loop as soon as the Number of cost Function Evaluations (NFE) reaches NFE_{max}
  - ◦ Update $X_{G_{best}}^t$
- If $i = N$ and $NFE < NFE_{max}$ return to Step 3, otherwise exit the loop and go to Step 4

Step 4. After termination of the main loop, announce the updated $X_{G_{best}}$ as the optimum solution.

Table 1: Unimodal benchmark functions

Function	Dim	Range	f_{min}
$f_1(x)=\sum_{i=1}^{n} x_i^2$	30	[-100,100]	0
$f_2(x)=\sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	[-10,10]	0
$f_3(x)=\sum_{i=1}^{n}\left(\sum_{j-1}^{i} x_j\right)^2$	30	[-100,100]	0
$f_4(x)=max_i\left\{\lvert x_i \rvert, 1 \le \mathrm{i} \le n\right\}$	30	[-100,100]	0
$f_5(x)=\sum_{i=1}^{n-1}\left(100\left(x_{i+1}-x_i^2\right)^2+\left(x_i-1\right)^2\right)$	30	[-30,30]	0
$f_6(x)=\sum_{i=1}^{n}\left(\left[x_i+0.5\right]\right)^2$	30	[-100,100]	0
$f_7(x)=\sum_{i=1}^{n} i x_i^4 + random\left[0,1\right)$	30	[-1.28,1.28]	0

4. OPTIMIZATION OF BENCHMARK FUNCTIONS

No-free-lunch theorem [23] states that no single algorithm is better than the others in all of the problems. In addition, meta-heuristic algorithms have stochastic nature so several test cases and trial runs are usually employed to compare their performance. The proposed method is evaluated on a number of unconstrained benchmarks including unimodal and multimodal test functions. The mathematical formulation, domain and true optimal solution of the treated functions are listed in Table 1 and Table 2. Two-dimensional plots of the functions are demonstrated in Fig. 2 and Fig. 3.

Table 2: Multimodal benchmark functions

Function	**Dim**	**Range**	f_{min}
$f_8(x)=\sum_{i=1}^{n}-x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$	30	[-500,500]	$-418.9829\times D$
$f_9(x)=\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	30	[-5.12,5.12]	0
$f_{10}(x)=-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)+20+e$	30	[-32,32]	0
$f_{11}(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	30	[-600,600]	0
$f_{12}(x)=\frac{\pi}{n}\left\{10\sin(\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2\left[1+10\sin^2(\pi y_{i+1})+(y_n-1)^2\right]\right\}+\sum_{i=1}^{n}u(x_i,10,100,4)$ $y_i=1+\frac{x_i+1}{4}$ $(x_i,a,k,m)=\begin{cases} k(x_i-a)^m & x_i>a \\ 0 & -a<x_i<a \\ k(-x_i-a)^m & x_i<-a \end{cases}$	30	[-50,50]	0
$f_{13}(x)=0.1\left\{\sin^2(3\pi x_1)+\sum_{i=1}^{n}(x_i-1)^2\left[1+\sin^2(3\pi x_i+1)\right]+(x_n-1)^2\left[1+\sin^2(2\pi x_n)\right]\right\}+\sum_{i=1}^{n}n(x_i,5,100,4)$	30	[-50,50]	0

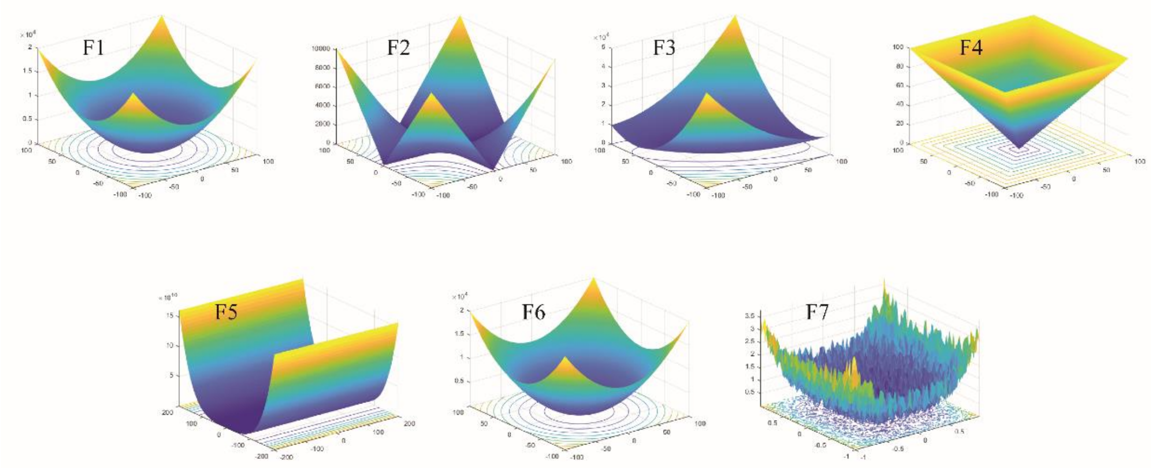

Figure 2. Two-dimensional illustration of the unimodal test functions

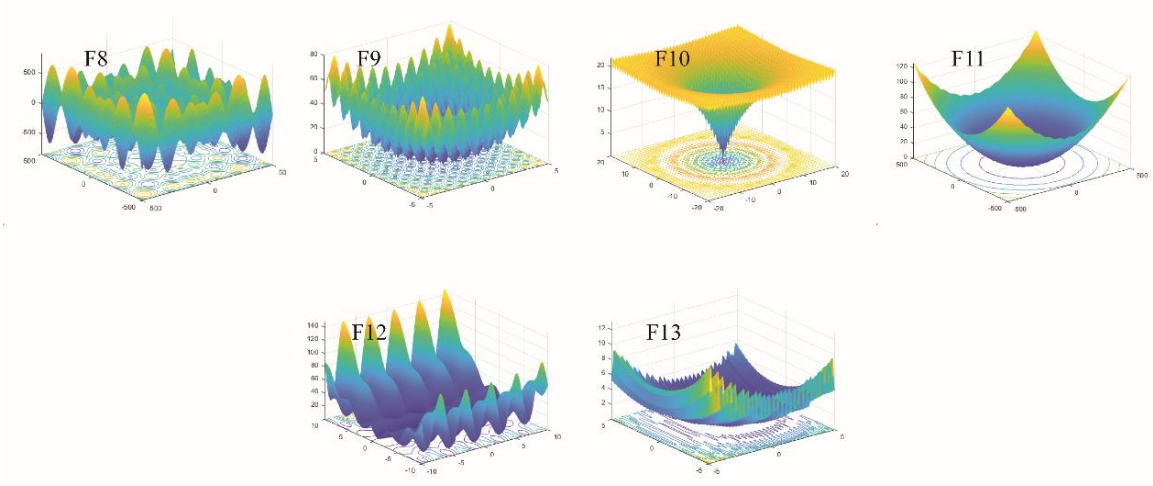

Figure 3. Two-dimensional illustration of the multimodal test functions

In addition to MSCA, a number of other meta-heuristics are added into the comparison including the standard Sine-Cosine Algorithm (SCA) [4], Particle Swarm Optimization (PSO) [14], Falcon Optimization Algorithm (FOA) [16], Marine Predators Algorithm (MPA) [19], Lightning Attachment Procedure Optimization (LAPO) [18], Multi-Verse Optimizer (MVO) [17], and Stochastic Paint Optimizer (SPO) [15]. The general parameters; $N = 20$ and $NFE_{max} = 5000$ are identically set for all the algorithms. The other control parameters are applied as reported in literature due to Table 3.

Table *4* This table gives the statistical results (mean and standard deviation); derived over 30 independent runs provided that at each run a new initial population is generated and shared between the algorithms. Superior results are bolded in the tables. The resulting convergence plots are given in Fig. 4. As the fair comparions are performed based on the same function evaluations; the algorithms with different FE_{ii} have ended in different iterations. FE_{ii} is a metric introduced by [24], to address the number of function evaluations for each individual at any iteration of the main loop in a population-based algorithm.

Table 3: Intrinsic parameters of the employed algorithms in addition to N and $NFE_{\max}$

MSCA	SCA	SPO	PSO	FOA	MVO	MPA	LAPO	BES	MRFO
$a = 0.5$ $b = 3$ $c = 1$ $d = 0.35$	$a = 2$	-	$C_{\text{Inertial}} = 1$ $C_{\text{cognitive}} = 2$ $C_{\text{social}} = 2$	$AP = 0.1$ $DP = 0.8$ $b = 1$ $\alpha = 0.1$ $f_c = 2$ $c_c = 2$ $c_s = 2$	$WEP_{\max} = 1$ $WEP_{\min} = 0.2$	FADs=0.2 P=0.5	-	$\alpha = 1$ a=10 R=1.5	-

Table 4: Results comparison for the test functions

Functions		MSCA	SCA	SPO	PSO	FOA	MVO	MPA	LAPO
F1	Best	**4.600E-20**	15.209993	7458.2091	37.575755	464.90974	3.1525194	1.164E-09	9.564E-07
	Mean	**5.999E-18**	594.67491	22668.03	328.36756	970.00979	7.9093825	453.06786	2.655E-06
	S.D.	**1.058E-17**	682.89674	8617.7788	204.06517	216.54965	2.6300001	1796.2853	1.198E-06
F2	Best	**2.054E-13**	0.0410206	27.193502	3.9284958	11.689133	0.8987685	5.124E-06	0.0002819
	Mean	**5.803E-12**	1.0525682	55.411075	9.6326811	14.737815	1.8745824	1.8147124	0.0006707
	S.D.	**5.915E-12**	1.3439008	17.570417	4.240935	1.8911275	0.7135218	4.739519	0.0001772
F3	Best	**6.671E-07**	3261.2057	18493.138	1553.5064	1809.1161	526.02582	0.1284737	0.0012567
	Mean	**0.0290714**	18938.344	52826.5	7687.7204	3727.2667	1491.1539	35858.98	0.0736287
	S.D.	**0.1167901**	9343.095	28854.413	4822.0889	1149.7085	665.56337	84096.079	0.1798566
F4	Best	**8.198E-06**	44.151322	37.337415	0.9538532	9.2115958	2.9747804	0.0001357	0.0006257
	Mean	**0.0010409**	59.210057	57.991271	5.6504879	14.037944	6.0405035	7.3114192	0.0014402
	S.D.	0.0035817	9.0460773	8.8499774	2.6793358	2.3549168	2.1380616	16.715435	**0.0004664**
F5	Best	**26.555777**	19623.189	8334752.4	191.03850	18433.465	117.4848	27.592315	28.564381
	Mean	**27.956081**	1894166.9	42528815	9114.6846	69767.723	539.82474	1339263.2	28.845898
	S.D.	0.7376787	2192527.5	23526755	13666.674	33071.186	491.85318	2661592.9	**0.0954093**
F6	Best	1.9875424	26.753514	6844.0796	36.962635	505.33744	3.6961081	**0.7033978**	4.2154523
	Mean	**2.6940189**	631.52855	24871.192	406.51821	896.94067	7.5768828	1243.1654	4.8562606
	S.D.	0.4182572	880.17643	9158.2112	275.36104	188.23177	2.1188144	2514.9044	**0.2905724**
F7	Best	**0.0085469**	0.2606019	1.1344287	0.1636464	0.1494028	0.0865105	0.0733587	0.010857
	Mean	**0.5352717**	1.2524237	9.8478646	0.6001167	0.5706649	0.6395721	1.2964520	0.5724748
	S.D.	0.2867277	1.0434461	10.834819	0.2940653	0.3253213	**0.2765063**	2.4281407	0.2976339
F8	Best	-7982.437	-4496.782	-6899.744	-8306.0393	-6494.948	**-9187.702**	-3085.675	-5678.458
	Mean	-6011.339	-3515.423	-5435.315	-6300.3948	-4505.116	**-7548.059**	-345.3484	-5549.597
	S.D.	952.23766	321.16892	771.01622	788.65618	770.32264	624.4339	1353.5274	**48.364647**
F9	Best	**4.547E-13**	1.8021127	69.031508	87.320333	104.32673	68.691141	4.841E-09	4.163E-06
	Mean	10.017938	86.493433	198.34621	173.67585	179.31987	117.98069	19.534238	**9.4551842**
	S.D.	**17.552821**	54.06053	49.863438	46.155547	27.001312	27.981337	47.075687	34.634397
F10	Best	**3.503E-11**	0.6743298	14.85608	2.7783469	6.3404485	2.1382609	1.115E-05	11.140617
	Mean	**5.85E-10**	13.437523	17.515196	4.9549361	7.6251011	2.8373253	2.4840073	12.986958
	S.D.	**5.001E-10**	7.8092472	1.2298877	1.1315359	0.6698406	0.4990059	4.9643508	0.934709
F11	Best	**0**	1.1382408	60.283315	1.0927022	5.7299923	1.0333528	1.934E-09	6.665E-07
	Mean	0.0063276	5.025946	225.0756	4.3074432	8.7056357	1.0698619	10.324249	**3.945E-06**
	S.D.	0.0131398	4.3342801	101.92668	2.6477378	1.8953664	0.0201534	20.264582	**2.318E-06**
F12	Best	0.0567657	4.7206038	349091.64	0.9996752	6.1794547	0.5164856	**0.0499558**	0.3558409
	Mean	**0.2401464**	3332941.2	53430869	3.3184937	12.116224	2.8615695	198872.96	0.5974686
	S.D.	0.1802676	6414652.4	53091857	1.4344056	3.4782643	1.3624701	769255.87	**0.1372273**
F13	Best	1.3460219	19.687762	4642121.3	4.4973541	26.014008	**0.2295851**	0.6674405	2.1470804
	Mean	1.8790132	6542617.7	11987325	67.171351	2926.044	**0.8047318**	1473974.7	2.6241069
	S.D.	0.2344503	12162043	79507449	250.21973	6364.4524	0.5376749	4572885.3	**0.2199844**

Superior results are bolded.

According to Fig. 4, MSCA has exhibited superior convergence rates over the others in most cases; especially for the first set of test functions with single global optimum. It is evident from the convergence plots of F8 to F13 that capability of MSCA in overpassing multiple local optima, is quite competitive with the other algorithms.

As reported in Table 4, superior mean results belong to MSCA for all the unimodal test functions. In the other cases (except for the F8 function), the proposed algorithm has obtained the first or the second rank among the others.

Table 5: Results of p-value comparison given by the Wilcoxon's statistical test.

F	MSCA vs. SCA	MSCA vs. SPO	MSCA vs. PSO	MSCA vs. FOA	MSCA vs. MVO	MSCA vs. MPA	MSCA vs. LAPO
F1	6.805E-43	2.415E-42	3.437E-33	7.146E-30	2.49E-39	4.533E-19	0.0965341
F2	4.293E-36	5.587E-34	8.039E-39	5.266E-38	1.349E-42	2.362E-13	6.478E-12
F3	1.346E-42	1.617E-42	1.537E-42	7.878E-20	1.615E-42	0.0007665	2.114E-20
F4	1.965E-42	2.681E-42	7.819E-25	8.873E-28	1.398E-42	2.964E-22	1.349E-21
F5	1.329E-42	2.83E-41	9.196E-38	1.203E-30	1.332E-42	0.0010715	0.1178195
F6	1.338E-42	1.382E-42	2.592E-36	2.503E-33	6.993E-41	2.691E-09	8.553E-05
F7	1.092E-42	1.467E-37	2.391E-38	7.198E-34	4.274E-40	1.962E-22	1.265E-21
F8	2.661E-43	3.765E-15	0.0028359	5.286E-44	1.581E-08	3.897E-20	2.535E-27
F9	1.047E-42	8.857E-42	2.375E-42	1.375E-41	5.839E-42	2.728E-21	2.501E-13
F10	1.347E-42	1.779E-42	4E-41	5.323E-42	1.349E-42	4.24E-19	3.134E-19
F11	1.049E-42	3.345E-37	2.521E-34	7.019E-29	1.347E-42	1.47E-20	0.0027535
F12	1.24E-42	7.841E-32	4.728E-33	3.703E-22	6.276E-39	7.156E-13	0.1923834
F13	1.293E-42	8.258E-32	7.173E-32	4.923E-24	1.931E-41	7.04E-09	0.0061478

Such numerical results are also validated by a non-parametric statistical experiment; called Wilcoxon rank-sum test. It is implemented between MSCA and every other algorithm.

According to Table 5, in most cases the resulting p-values have fallen well below the threshold of 0.05. It reveals superiority of MSCA over the other methods by 95% confidence (P-values less than 5%); in most of the treated cases except comparison of MSCA vs. PSO in solution of F8, vs. MPA in solution of F3 and F5 and vs. LAPO in solution of F1, F5 and F12. In another word, MSCA has been quite superior to the other methods in the 85 out of 91 total cases. The matter confirms significant confidence for statistical difference of the proposed MSCA with respect to SCA and the other treated algorithms.

5. OPTIMIZATION OF CONSTRAINED PROBLEMS

The performance of the proposed algorithm is further evaluated on a number of constrained engineering and structural problems. Each problem has some inequality constraints of the form $g(X) \leq 0$ where X stands for the design vector. Since the algorithms exclusively handle single objective functions, the constraints' effect is considered using the following external penalty formula.

$$\min Cost(X) = f(X) \times (1 + K_P . \sum_l \max(g_l, 0)) \tag{7}$$

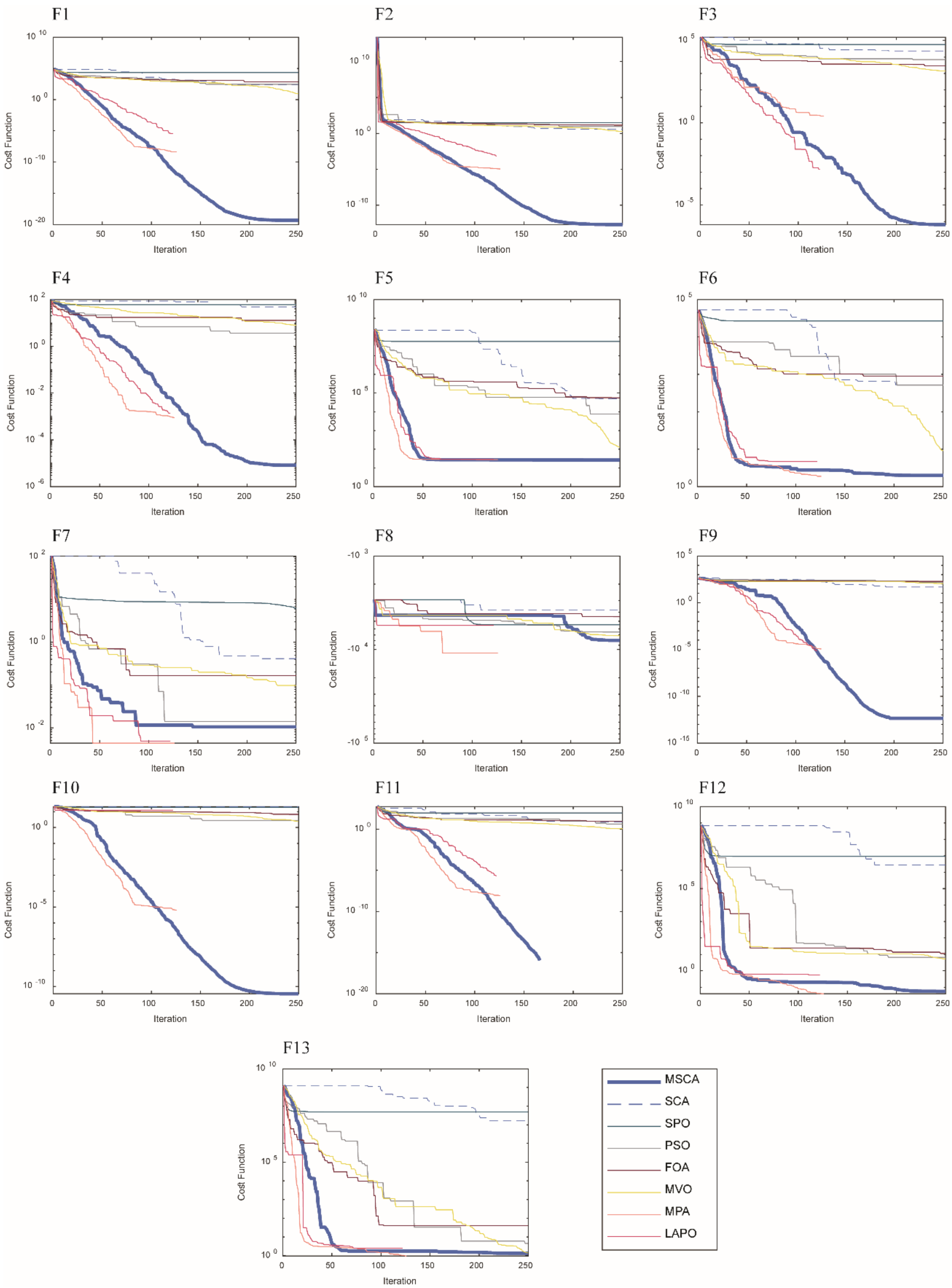

Figure 4. Convergence comparison of MSCA with the other algorithms in solving test functions

Where $g_l(X)$ stands for value of the l^{th} inequality constraint and K_p is the penalty factor. The raw and penalized objective functions are denoted by $f(X)$ and $Cost(X)$, respectively. During optimization, the values of the variables are enforced to fall within their prescribed bounds. In each case, 20 individual search agents are employed while the penalty factor is taken 1000.

5.1 Engineering problems

In this subsection, six engineering problems in a variety of discrete, continuous and mixed discrete-continuous types are solved by 30 independent runs. Every problem undergoes two sets of comparisons. First: the best result of MSCA is compared with those previously reported in the literature. Second: MSCA and nine other meta-heuristic algorithms are programmed and run for each problem under the fair comparison conditions as introduced by Shahrouzi et al. [24]. Under such conditions all the methods spend the same number of function calls; stating from identical initial population at each run while the same constraint handling is also applied.

In this problems, several statistical measures are derived including the best, worst and mean cost values followed by Standard Deviation (SD), Coefficient of Variation (CV) and a Variation Index (VI). The latter is defined in [24,25] for fair comparison.

In addition to MSCA, SCA, SPO, PSO, FOA, MVO, MPA, LAPO, two recent meta-heuristics; i.e. Bald Eagle Search (BES) [20] and Manta Ray Foraging Optimization (MRFO) [21], are selected for the 2nd set of comparisons. Formulation of each engineering problem is given in Appendix A.

5.1.1 The tension-compression spring problem

In this problem the volume of the spring in Fig. 5, is to be minimized as a non-linear objective function subject to one linear and three non-linear constraints. They limit the deflection, the shear stress, and the surge frequency. The problem involves three continuous design variables including the wire diameter (d or x_1), mean coil diameter (D or x_2), and the number of active coils (N or x_3). Table 6 reveals that MSCA has captured the best result with lower computational effort than Manta Ray Foraging Optimization (MRFO) [21], Smart Flower Optimization Algorithm (SFOA) [26], and Zebra Optimization Algorithm (ZOA) [27].

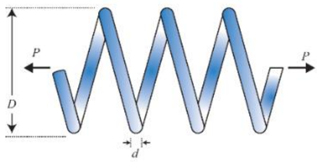

Figure 5. The tension/compression spring

Since comparison with literature works in Table 6, does not guarantee identical test conditions, the second experiment is performed under the fair-comparison conditions and the results are reported in Table 7. It is observed that the best result belongs to MSCA and

MPA; while MSCA has also better mean and standard deviation. In this example, FOA, SCA and MSCA are among the most robust methods regarding the statistical measures. Fig. 6 shows high convergence rate of MSCA among the treated meta-heuristics. It also reveals that despite MSCA, some other methods have been trapped in local optima.

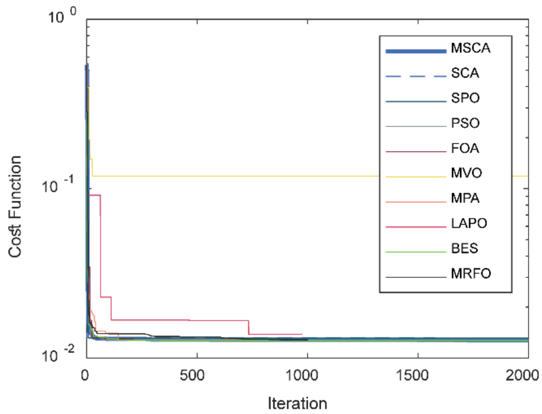

Figure 6. Convergence histories of the tension/compression spring at the best design.

Table 6: The results of MSCA vs. those of the literature for tension-compression spring problem

Algorithm	NFE	x_1	x_2	x_3	Best	Mean	SD	VI
MSCA	40,000	0.051644	0.355626	11.353256	**0.0126653**	0.0128126	2.962E-04	2.77E+01
MRFO [21]	50,000	0.0523734	0.3733461	10.3831265	0.0126757	0.0127007	2.137E−04	2.52E+01
SFOA [26]	100,000	0.0517943	0.359256	11.1416	0.01268410	0.0127312	6.767E−05	1.59E+01
ZOA [27]	100,000	0.0520983	0.366644	10.7299	0.01266801	0.0126812	0.000023	5.441+00

Table 7: Results of the present work for the tension-compression spring problem

Algorithm	MSCA	SCA	SPO	PSO	FOA	MVO	MPA	LAPO	BES	MRFO
Best	**0.012665**	0.012762	0.012685	0.012679	0.012692	0.012796	**0.012665**	0.013814	0.012674	0.012685
Mean	0.012812	0.012974	0.023619	0.014366	0.012767	0.016999	0.014124	0.015840	0.020038	0.014496
Worst	0.014229	0.013225	0.109855	0.030460	0.013082	0.018003	0.016980	0.018455	0.109616	0.028774
Infeasibility	0	0	0	0	0	0	0	0	0	0
SD	2.962E-04	1.19E-04	2.253E-02	4.644E-03	8.176E-05	1.648E-03	1.872E-03	1.49E-03	2.111E-02	4.331E-03
CV	2.31E-02	9.20E-03	9.54E-01	3.23E-01	6.40E-03	9.69E-02	1.33E-01	9.46E-02	1.05E+00	2.99E-01
VI	2.77E+01	1.10E+01	1.14E+03	3.88E+02	7.68E+00	1.16E+02	1.59E+02	1.14E+02	1.26E+03	3.59E+02
x_1	0.051644	0.053272	0.050640	0.051360	0.050817	0.050243	0.051686	0.054469	0.052065	0.052161
x_2	0.355626	0.395505	0.332008	0.348652	0.335989	0.322894	0.356645	0.416237	0.365763	0.368088
x_3	11.353256	9.370574	12.899581	11.786368	12.628008	13.699092	11.293250	9.186824	10.783405	10.666432

5.1.2 The speed reducer problem

The objective of the speed reducer issue is to minimize the volume (a nonlinear function) while taking into account a variety of eleven constraints. Among these constraints, four are linear constraints and seven are nonlinear. These constraints are imposed on the bending stress of the gear teeth, the surface stress, the transverse deflections of the shafts, and the stresses in the shafts illustrated in Fig. 7. The problem involves seven design variables, which are of both discrete and continuous types. They include the face width b (x_1), module of the teeth m (x_2), the integer number of the teeth in the pinion z (x_3), the length of the first shaft between bearings l_1 (x_4), the length of the second shaft between bearings l_2 (x_5), the diameter of the first shaft d_1 (x_6), and the diameter of the second shaft d_2 (x_7). Table 8 reports that MSCA by spend 40000 fitness evaluations to capture the cost of 2994.6028; that is lower than the other results by Arithmetic Optimization Algorithm (AOA) [28], Passing Vehicle Search (PVS) [29] and ZOA [27].

Table 8: The results of MSCA vs. those of the literature for the speed reducer problem

Algorithm	**NFE**	x_1	x_2	x_3	x_4	x_5	x_6	x_7	**Best**	**Mean**	**SD**	**VI**
MSCA	40,000	3.500002	0.7	17	7.300902	7.720612	3.350237	5.286656	**2994.6028**	3001.8393	5.178E+00	2.07E+00
AOA [28]	30,000	3.4976	0.7	17	7.3	7.8	3.3501	5.2857	3.00E+03	3.00E+03	1.22E-12	3.66E-13
PVS [29]	54,350	3.5	0.7	17	7.3	7.8	3.35021	5.28668	2996.3481	2996.3481	0	0
ZOA [27]	100,000	3.50112	0.7	17	7.3423	7.85116	3.35194	5.28818	2998.5189	2999.212	1.36421	1.36E+00

Table 9: Results of the present work for the speed reducer problem

Algorithm	**MSCA**	**SCA**	**SPO**	**PSO**	**FOA**	**MVO**	**MPA**	**LAPO**	**BES**	**MRFO**
Best	**2994.6028**	3055.8173	3053.4758	3001.7287	2996.9302	2998.7109	3005.5754	3013.9894	2994.6033	3238.8845
Mean	3001.8393	3109.8093	3644.3513	3116.3919	3018.8776	3008.1942	3371.4744	3110.1163	3028.6383	55190.8750
Worst	3016.4200	3164.8770	5800.7796	3355.5595	3165.3647	3023.1556	5482.7257	3598.0878	3161.8431	146609.182
Infeasibility	0	0	0	0	0	0	0	0	0	0
SD	5.178E+00	3.032E+01	7.986E+02	9.712E+01	3.248E+01	5.853E+00	8.245E+02	1.651E+02	5.177E+00	3.686E+04
CV	1.72E-03	9.75E-03	2.19E-01	3.12E-02	1.08E-02	1.95E-03	2.45E-01	5.31E-02	1.71E-02	6.68E-01
VI	2.07E+00	1.17E+01	2.63E+02	3.74E+01	1.29E+01	2.33E+00	2.93E+02	6.37E+01	2.05E+01	8.01E+02
x_1	3.500002	3.506199	3.600000	3.503236	3.500000	3.500052	3.505879	3.504897	3.500025	14.936715
x_2	0.700000	0.700000	0.700000	0.700000	0.700000	0.700000	0.700000	0.700000	0.700004	0.703075
x_3	17.000000	17.000000	17.000000	17.000000	17.000000	17.000000	17.000000	17.000000	17.000000	17.232117
x_4	7.300902	7.300000	8.299528	7.307484	7.400000	7.300000	7.491783	7.346492	7.303018	38.151256
x_5	7.720612	7.828030	8.121004	7.879732	7.726512	7.811277	7.733214	7.822444	7.718183	7.942728
x_6	3.350237	3.513268	3.352388	3.356323	3.351390	3.357486	3.361421	3.406666	3.350260	-7.946850
x_7	5.286656	5.306784	5.288926	5.287836	5.288278	5.287057	5.292685	5.286960	5.286657	-3.984415

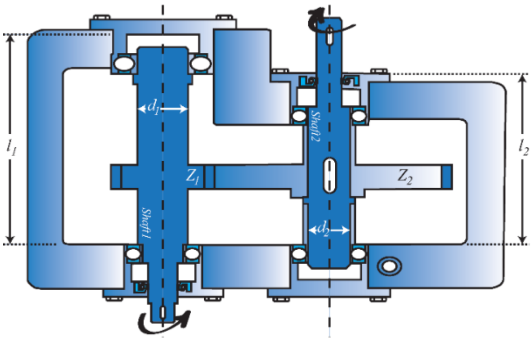

Figure 7. The speed reducer

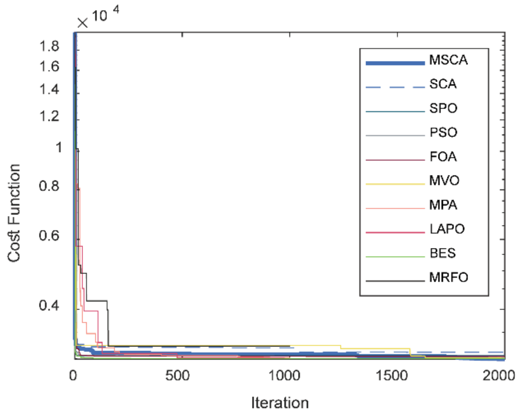

Figure 8. Convergence histories of the speed reducer at the best design

Implementing a fair comparison test, MSCA has outperformed the other nine meta-heuristics in terms of the best and mean results, as reported in Table 9. After MSCA (with the best cost of 2994.6028), BES has stood on the second rank by revealing 2994.6033; however, with slightly lower VI of 20.5. Comparison of the convergence rates in Fig. 8 shows that MSCA, BES, PSO and LAPO have been more efficient than the others in this problem.

5.1.3 The welded beam design problem

The objective of the welded beam problem (illustrated in Fig. 9) is to minimize the manufacturing cost (a nonlinear function) subject to seven inequality constraints. These constraints pertain to the shear stress, the bending stress in the beam, the buckling load, and the beam deflection. The problem involves four continuous design variables, i.e. the welding dimensions h, l (x_1, x_2) in addition to the beam cross-sectional height t (x_3) and width b (x_4) .

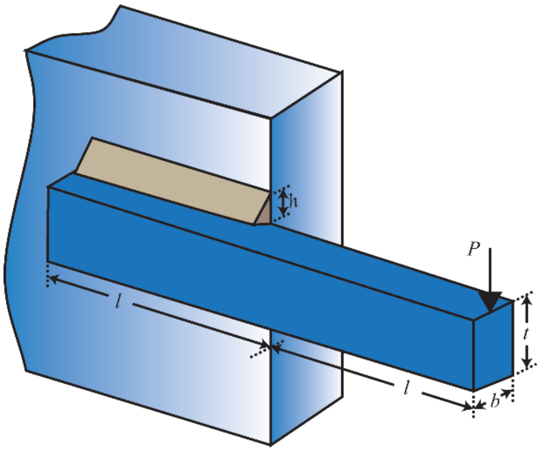

Figure 9. The welded beam design

According to Table 10, MSCA has captured the best design of this problem (1.724853) in comparison with Water Cycle Algorithm (WCA) [30], Firefly Algorithm (FA) [31] and ZOA [27]. WCA has achieved 1.724856 with 46450 fitness evaluations; that is 55% higher than 30000 fitness evaluations by MSCA.

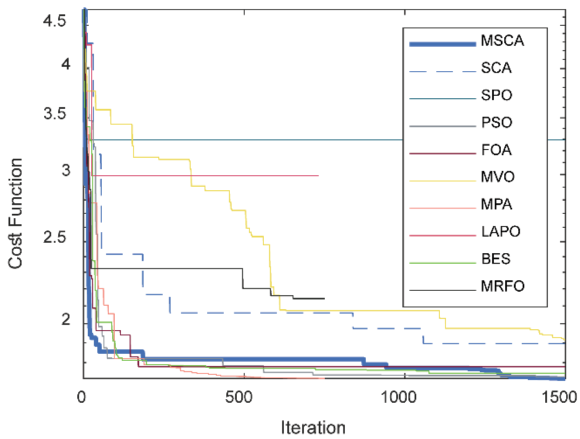

Figure 10. Convergence histories of the welded beam design at the best design

Table 10: The results of MSCA vs. those of the literature for the welded beam problem

Algorithm	**NFE**	x_1	x_2	x_3	x_4	**Best**	**Mean**	**SD**	**VI**
MSCA	30,000	0.205729	3.470500	9.036630	0.205730	**1.724853**	1.726152	2.092E-03	1.09E+00
WCA [30]	46,450	0.205728	3.470522	9.036620	0.205729	1.724856	1.726427	4.29E-03	2.89E+01
FA [31]	50,000	0.2015	3.562	9.0414	0.2057	1.73121	1.878656	0.2677989	7.13E+02
ZOA [27]	100,000	0.205739	3.470261	9.036623	0.205740	1.724916	1.725326	0.000010	1.73E-02

Table 11: Results of the present work for the welded beam problem

Algorithm	MSCA	SCA	SPO	PSO	FOA	MVO	MPA	LAPO	BES	MRFO
Best	**1.72485**	1.81654	1.74804	1.73268	1.74387	1.72849	1.72493	2.45080	1.73719	**1.72485**
Mean	1.72615	1.87643	2.62976	1.78129	1.76537	1.76230	1.76950	3.17811	2.09278	84.44331
Worst	1.73203	1.93744	4.69661	2.32523	1.79385	1.91515	1.89373	4.03747	4.03140	809.8500
Infeasibility	0	0	0	0	0	0	0	0	0	0
SD	2.094E-03	3.462E-02	7.611E-01	1.075E-01	1.226E-02	4.753E-02	6.882E-02	4.202E-01	5.413E-01	2.425E+02
CV	1.21E-03	1.84E-02	2.89E-01	6.03E-02	6.94E-03	2.70E-02	3.89E-02	1.32E-01	2.59E-01	2.87E+00
VI	1.09E+00	1.66E+01	2.60E+02	5.43E+01	6.25E+00	2.43E+01	3.50E+01	1.19E+02	2.33E+02	2.58E+03
x_1	0.205729	0.190480	0.198980	0.202427	0.201389	0.203406	0.205731	0.196676	0.203632	0.205730
x_2	3.470500	4.157950	3.557217	3.537500	3.516711	3.522208	3.470662	5.119694	3.559438	3.470490
x_3	9.036630	8.964726	9.199641	9.050680	9.165716	9.038457	9.036807	7.684496	9.054398	9.036624
x_4	0.205730	0.210676	0.204930	0.205931	0.205369	0.205727	0.205732	0.315768	0.205797	0.205730

Another experiment is performed under fair-comparison conditions with nine other meta-heuristics. As reported in Table 11, MSCA has captured the best result among the others. MRFO has revealed similar best result but with very higher mean value and standard deviation (or VI) than MSCA. Hence, MSCA is on the first rank of this test in both quality and robustness. It has also been more efficient than the others regarding the convergence curves in Fig. 10.

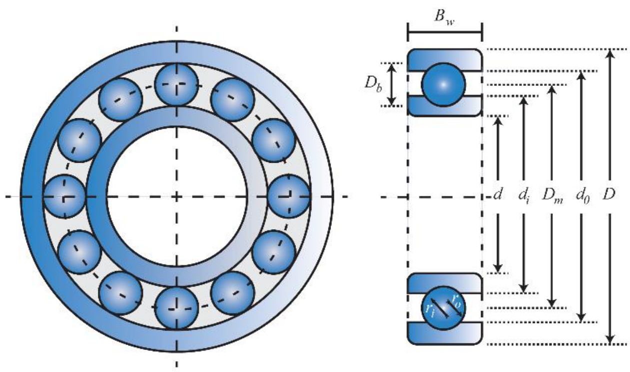

Figure 11. The rolling element bearing

5.1.4 The rolling element bearing problem

In optimal design of the rolling element bearing (Fig. 11), the objective is to optimize the dynamic load carrying capacity. This optimization problem is subject to nine inequality constraints that pertain to the kinematic conditions and manufacturing requirements. There are ten design variables in such a mixed continuous-discrete problem; including the pitch diameter D_m (x_1), the ball diameter D_b (x_2), the number of balls Z (x_3), the inner and outer raceway curvature coefficients f_i, f_o (x_4, x_5), as well as five other variables K_{Dmax}, K_{Dmin}, ε, e, ζ (x_6: x_{10}) that impact the geometry of the bearing.

Table 12 compares MSCA in solution of this problem with a number of other previous works. Although such comparison is not done under fair conditions, it declares that MSCA has been successful in capturing the best (global) optimum; already reported in literature.

Table 12: The results of MSCA vs. those of the literature for the rolling element bearing problem

Algorithm	NFE	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	Best	Mean	SD	VI
MSCA	60,000	125.719	21.426	11	0.515	0.515	0.462	0.696	0.30	0.026	0.621	**-81859.74**	**-81781.48**	144.574	3.1820
GA [32]	225,000	125.717	21.423	11	0.515	0.515	0.4159	0.651	0.30	0.022	0.751	-81843.30	NA	NA	NA
PSO [33]	30,000	125	20.753	11.17	0.515	0.515	0.5	0.615	0.3	0.051	0.6	-81691.20	-50435.01	13962.1	2.5E+2
TLBO [34]	10,000	125.719	21.425	11	0.515	0.515	0.4242	0.633	0.3	0.069	0.799	-81859.74	-81438.98	0.66	8.1E-03

Under the fair comparison conditions, superiority of MSCA is declared over the other nine methods of Table 13, not only in the best but also in the mean results. In addition, MSCA has created the best variation index (VI) of 6.5. According to Fig. 12, the MSCA and BES algorithm have higher convergence rates than the other meta-heuristics in solution of this example.

Table 13: Results of the present work for the rolling element bearing problem

Algorithm	MSCA	SCA	SPO	PSO	FOA	MVO	MPA	LAPO	BES	MRFO
Best	**-81859.74**	-80642.08	-69859.27	-78884.58	-81827.12	-81842.92	-81379.11	-55970.55	-81849.56	-81845.78
Mean	-81781.48	-75395.42	-40439.29	-37572.99	-65673.55	-81373.24	-65464.05	-36390.27	-71322.04	-64391.72
Worst	-81392.24	-68059.42	-17341.61	-17836.58	-43338.92	-79794.29	-49548.99	-30134.68	-32023.17	-57383.26
Infeasibility	0	0	0	0	0	0	0	0	0	0
SD	144.5745	3560.556	16399.62	16948.57	19097.82	465.7585	22507.29	6422.268	16562.85	9951.161
CV	0.001768	0.047225	0.405537	0.451084	0.290799	0.005724	0.343812	0.176483	0.232226	0.154541
VI	3.182068	85.00518	729.9661	811.9511	523.4387	10.30271	618.8608	317.6696	418.0072	278.1738
x_1	125.7190	125.0000	125.0000	125.0006	125.7138	125.7166	125.5853	125.0000	125.7170	125.7102
x_2	21.42559	21.24940	19.67601	20.99844	21.42105	21.42319	21.38101	18.78230	21.42476	21.42370
x_3	11.00000	11.00000	10.82497	11.00000	11.00000	11.00000	11.00000	9.000000	11.00000	10.70851
x_4	0.515000	0.515000	0.515000	0.515000	0.515000	0.515000	0.515041	0.515000	0.515001	0.515000
x_5	0.515000	0.515000	0.515000	0.515008	0.515002	0.515000	0.515316	0.515000	0.515010	0.515001
x_6	0.462368	0.500000	0.400000	0.499999	0.466866	0.400436	0.500000	0.400000	0.495555	0.086109
x_7	0.695630	0.692185	0.695741	0.600004	0.665036	0.623592	0.689857	0.600000	0.629172	203.8183
x_8	0.300000	0.300000	0.351997	0.300000	0.300000	0.300000	0.302295	0.300000	0.300072	0.300269
x_9	0.025593	0.065781	0.100000	0.020039	0.020000	0.099646	0.100000	0.020000	0.099040	2.174563
x_{10}	0.621255	0.600000	0.600000	0.600004	0.657348	0.631677	0.669164	0.600000	0.711410	0.619952

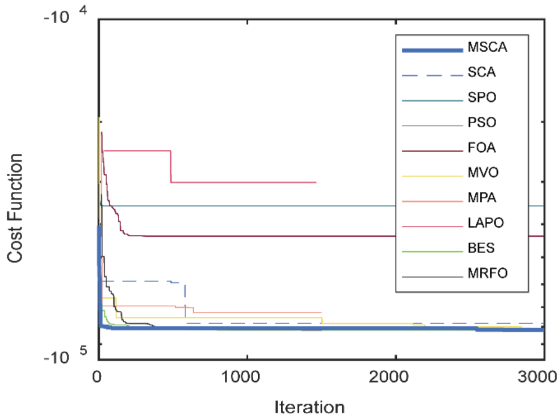

Figure 12. Convergence histories of the rolling element bearing at the best design

5.1.5 The coupling with bolted rim problem

The problem of designing the coupling with bolt rim (Fig. 13), is formulated to minimize the radius, the number of bolts, and lower torque of the coupling, subject to eleven inequality constraints. It is aimed to design a coupling with N (x_1) number of bolts placed at $R_b(x_2)$ radius, having a diameter d (x_3) that transmits a torque M (x_4) by adhesion. The variable d is discrete and N is an integer while R_b and M take continuous values.

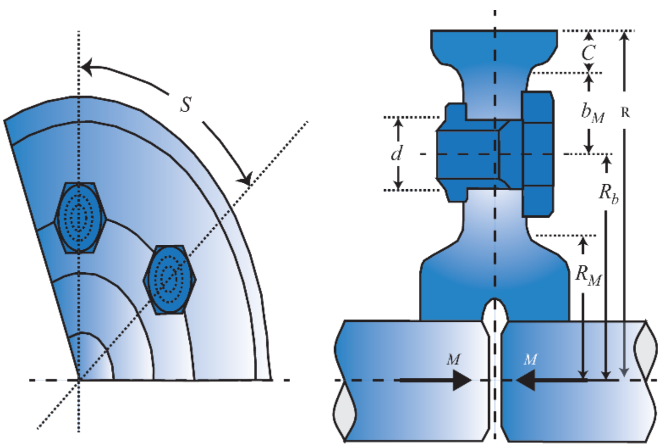

Figure 13. The coupling with bolted rim

According to Table 14, the proposed method has obtained 3.25 as the best cost; showing 14% improvement with respect to the result of 3.48 among the Rao Algorithm (Rao-3) [35], ABC [36] and Mine Blast Algorithm (MBA) [36]. Such a best result is obtained via just 500 fitness evaluations by MSCA; that is much lower than 5000 by the others. It has also obtained lower mean and VI; that confirms its superior robustness.

Table 15 lists the fair comparison results between 10 algorithms. Most of them (except MRFO) have captured the best result of 3.25; however, MSCA, MVO and LAPO have produced better mean results than the others. These three methods have also been superior to the others in view of robustness as they have obtained the least standard deviation.

According to Fig. 14, MSCA, SPO and BES have higher convergence rates among the treated algorithms.

Table 14: The results of MSCA vs. those of the literature for the coupling with bolted rim problem

Algorithm	**NFE**	x_1	x_2	x_3	x_4	**Best**	**Mean**	**SD**	**VI**
MSCA	500	6	8	50	40	3.25	**3.25**	0	0
Rao-3 [35]	5,000	6	8	57.5	40	3.40	3.40	3.140E-15	2.31E-13
ABC [36]	5,000	2	8	59.5	40	3.48	3.48	5.874E-06	4.22E-04
MBA [36]	5,000	2	8	59.5	40	3.48	3.48	7.415E-08	5.33E-06

Table 15: Results of the present work for the coupling with bolted rim problem

Algorithm	**MSCA**	**SCA**	**SPO**	**PSO**	**FOA**	**MVO**	**MPA**	**LAPO**	**BES**	**MRFO**
Best	**3.250000**	**3.250000**	**3.250000**	**3.250000**	**3.250000**	**3.250000**	**3.250000**	**3.250000**	**3.250000**	3.282189
Mean	**3.250000**	3.254167	3.671909	3.502537	3.261110	**3.250000**	3.448431	**3.250000**	3.346747	4.146859
Worst	3.250000	3.375000	4.750000	4.750000	3.447497	3.250000	4.503937	3.250000	4.280026	8.319461
Infeasibility	0	0	0	0	0	0	0	0	0	0
SD	0.000E+00	2.282E-02	4.698E-01	4.884E-01	4.305E-02	0.000E+00	2.926E-01	0.000E+00	2.413E-01	1.119E+00
CV	0.00E+00	7.01E-03	1.28E-01	1.39E-01	1.32E-02	0.00E+00	8.49E-02	0.00E+00	7.21E-02	2.70E-01
VI	0.00E+00	1.05E-01	1.92E+00	2.09E+00	1.98E-01	0.00E+00	1.27E+00	0.00E+00	1.08E+00	4.05E+00
x_1	6.000000	7.000000	6.000000	6.000000	6.000000	7.000000	7.000000	6.000000	6.000000	7.492532
x_2	8.000000	8.000000	8.000000	8.000000	8.000000	8.000000	8.000000	8.000000	8.000000	8.432537
x_3	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000	50.926236
x_4	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000	40.0000	40.5466

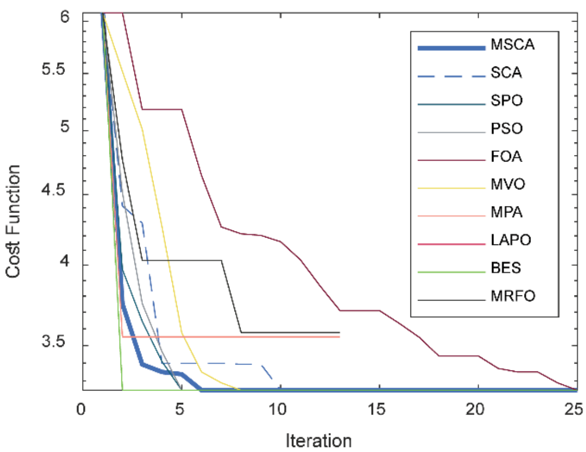

Figure 14. Convergence histories of the coupling with bolted rim at the best design.

5.1.6 The spur gear problem

The spur gear is to be designed due to the American Gear Manufacturers Association

(AGMA) standards. The problem is formulated to minimize the total weight of the gear, subject to eight inequality constraints; including the tooth bending strength, the tension strength of the shafts, and the gear dimensions, as shown in Fig. 15. The design variables include the face width of the tooth gear b (x_1), the diameter of the pinion d_1 (x_2), the diameter of the wheel shaft d_2 (x_3), the number of teeth of the pinion Z_1 (x_4), the module m (x_5), and the hardness in gear design H (x_6).

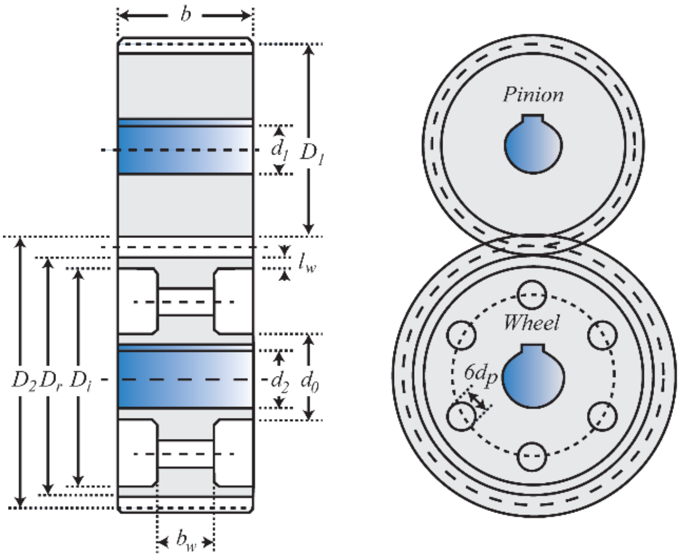

Figure 15. The spur gear

Table 16 reports solution of this problem by recent literature works; as well as by the proposed MSCA. It can be noticed that MSCA has been successful in capturing the best result reported in litureatue; i.e. 1538.9446. It is 6% lower than the optimal design by Atom Search Optimization (ASO) [37] and 48% lower than the best result of Artificial Algae Algorithm (AAA) [38].

Table 16: The results of MSCA vs. those of the literature for the spur gear problem

Algorithm	NFE	x_1	x_2	x_3	x_4	x_5	x_6	Best	Mean	SD	VI
MSCA	25,000	26.893776	30	17.174963	18	2.426107	400	**1538.9446**	1538.9456	1.17E-03	5.70E-04
ASO [37]	25,000	28.07787	27.91758	19.67367	18	2	392	1624.2236	1713.7505	49.413599	7.21E+01
AAA [38]	6,118	22	30	30	18	2	300.6048	2958.339	2958.339	8.45E-05	5.24E-06

The second experiment is performed under fair-comparison conditions between the meta-heuristic algorithms. According to Table 17, MSCA has earned the first rank regarding the best and mean results; with respect to SCA, SPO, PSO, FOA, MVO, MPA, LAPO, BES and MRFO. Superiority of MSCA in the mean results, ranges from 0.02 to 48 percent versus different algorithms. MSCA has also shown better robustness. Its standard deviation; i.e. 0.00117 is well below 0.211 by MVO (the least value among the other methods).

Table 17: Results of the present work for the spur gear problem

Algorithm	MSCA	SCA	SPO	PSO	FOA	MVO	MPA	LAPO	BES	MRFO
Best	**1538.9446**	1562.9965	1540.3134	1539.5944	1539.0796	1539.0228	1578.6290	1745.1009	1539.3921	1539.0013
Mean	1538.9456	1618.8044	3103.0132	1734.2182	1539.7595	1539.3137	2856.0856	2153.4708	2184.3084	2924.7153
Worst	1538.9488	1679.1977	16260.5301	6059.7526	1540.7602	1539.7798	5413.2792	3193.6970	16066.3669	6915.4423
Infeasibility	0	0	0	0	0	0	0	0	0	0
SD	1.170E-03	3.257E+01	2.982E+03	8.195E+02	4.713E-01	2.110E-01	1.504E+03	3.283E+02	2.751E+03	1.987E+03
CV	7.60E-07	2.01E-02	9.61E-01	4.73E-01	3.06E-04	1.37E-04	5.27E-01	1.52E-01	1.26E+00	6.79E-01
VI	5.70E-04	1.51E+01	7.21E+02	3.54E+02	2.30E-01	1.03E-01	3.95E+02	1.14E+02	9.45E+02	5.10E+02
x_1	26.893776	27.260658	26.924754	26.895013	26.894760	26.894062	27.300580	27.244108	26.894880	26.894530
x_2	30.000000	30.000000	30.000000	29.999876	30.000000	29.998204	29.108882	30.000000	30.000000	29.999999
x_3	17.174963	18.520902	17.202007	17.254182	17.187851	17.181428	19.196013	24.750611	17.228377	17.178690
x_4	18.0000	18.0000	18.0875	18.0000	18.0000	18.0000	18.0000	19.0000	18.0000	18.3248
x_5	2.426107	2.338667	2.153070	2.154390	2.680237	2.309650	2.061632	2.096723	2.657817	2.212042
x_6	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	399.695192

In this problem, Fig. 16 reveals that some algorithms like PSO and SPO have been trapped in local optima and some others like SCA has suffered from lack of search refinement in the final iterations. In the other hand, the proposed MSCA has exhibited stable convergence toward the global optimum.

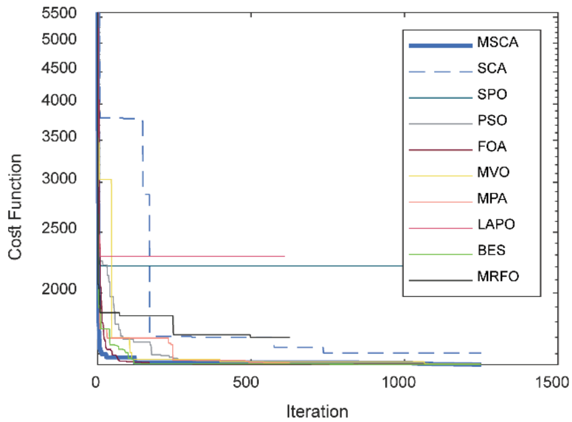

Figure 16. Convergence histories of the spur gear at the best design

5.2 Structural examples

In this subsection, two structural examples are provided to evaluate the performance of the MSCA in size optimization. Besides conducting a fair comparison between the MSCA and SCA algorithms, the results are also compared to other optimization techniques. In these problems, the number of runs is set to 30.

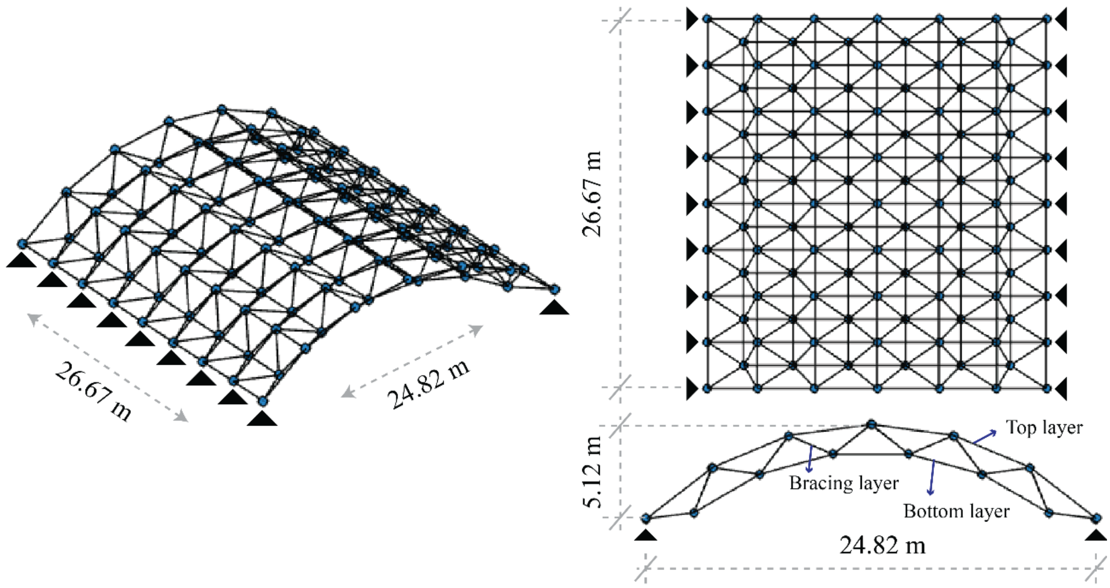

Figure 17. The 384-bar double layer barrel vault

5.2.1 The 384-bar braced barrel vault

In this example, a double-layer barrel vault is presented as a practical structure. The model of the 384-bar truss is illustrated in Fig. 17. The structure has been modeled based on the following specifications. The material density is 0.288 lb/in^3, the modulus of elasticity is 30450 ksi, the yield stress of the steel material is 58 ksi, and nodal displacements are limited to 0.1969 $inch$ in each of the x, y, and z directions. Concentrated loads of 20 $kips$ are applied downward to all free nodes in the top layer. Stress limits are enforced based on Eq. (7) and Eq. (8). In accordance with AISC regulations, the maximum slenderness ratio for tension members is restricted to 300, and for compression members, it is limited to 200. The cross section areas are selected from the discrete list of Table 18.

$$\sigma^{+} = 0.6F_y \tag{7}$$

$$\sigma^{-} = \begin{cases} \dfrac{\left(F_y\left[1-\dfrac{\lambda^2}{2C_c^2}\right]\right)}{\left(\dfrac{5}{3}+\dfrac{3\lambda}{8C_c}-\dfrac{\lambda^3}{8C_c^3}\right)} & \lambda < C_c \\ \dfrac{12\pi^2 E}{23\lambda^2} & \lambda \geq C_c \end{cases} \tag{8}$$

where E is the Young's modulus, F_y stands for the yield strength, $C_c = \sqrt{2\pi^2 E/F_y}$ denotes the critical slenderness ratio, and $\lambda = kL/r$ represents the maximum slenderness ratio. The effective length factor, the member length, and the section's radius of gyration are denoted by k, L and r, respectively.

Table 18: The steel pipe sections

No.	**Type**		**Area (in^2)**	**Gyration radius (in)**	**No.**	**Type**		**Area (in^2)**	**Gyration radius (in)**
1	ST	1/2	0.25	0.2608	20	EST	3 1/2	3.68	1.3063
2	EST	1/2	0.32	0.25	21	DEST	2 1/2	4.03	0.8439
3	ST	3/4	0.33	0.3333	22	ST	5	4.3	1.8801
4	EST	3/4	0.43	0.3224	23	EST	4	4.41	1.4762
5	ST	1	0.49	0.4197	24	DEST	3	5.47	1.0465
6	EST	1	0.64	0.4073	25	ST	6	5.58	2.2441
7	ST	1 1/4	0.67	0.5399	26	EST	5	6.11	1.8406
8	ST	1 1/2	0.8	0.6229	27	DEST	4	8.1	1.3744
9	EST	1 1/4	0.88	0.5241	28	ST	8	8.4	2.9378
10	EST	1 1/2	1.07	0.7889	29	EST	6	8.4	2.1958
11	ST	2	1.07	0.6045	30	DEST	5	11.3	1.7244
12	EST	2	1.48	0.7658	31	ST	10	11.9	3.6782
13	ST	2 1/2	1.7	0.9515	32	EST	8	12.8	2.8777
14	ST	3	2.23	1.1637	33	ST	12	14.6	4.3715
15	EST	2 1/2	2.25	0.9238	34	DEST	6	15.6	2.0616
16	DEST	2	2.66	0.7018	35	EST	10	16.1	3.6287
17	ST	3 1/2	2.68	1.3369	36	EST	12	19.2	4.3421
18	EST	3	3.02	1.1349	37	DEST	8	21.3	2.7578
19	ST	4	3.17	1.5102	-				

In this example, as per Table 19, the modified sine-cosine algorithm (MSCA) achieved the best result of 61,421.95 lb, while the sine-cosine algorithm (SCA) attained a best result of 95,281.52 lb after 40,000 evaluations of the objective function. The 43.21% improvement in response demonstrates the effectiveness of the modified version in this problem. In addition, the modified sine-cosine algorithm (MSCA) has outperformed the algorithms documented in Table 19. The convergence curve displayed in Fig. 18 clearly illustrates the enhanced performance and rapid convergence speed of the modified version in comparison to the original version. As Fig. 19, both the displacements and stresses fall within the permissible range, with zero infeasibility.

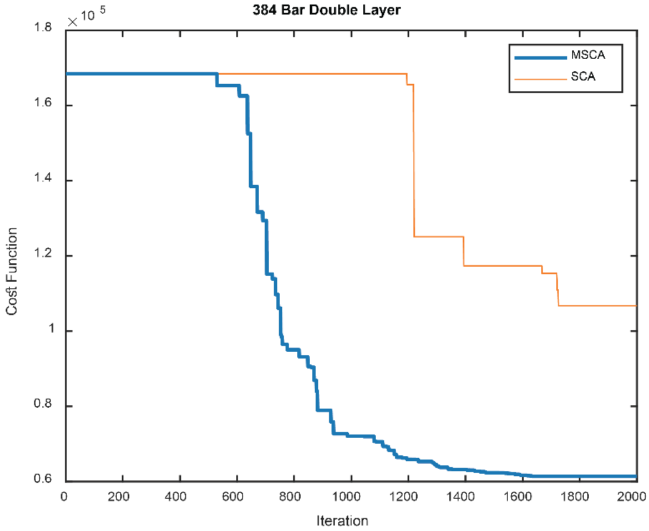

Figure 18. Convergence curve of the 384-bar double layer barrel vault

Table 19: Comparison of the optimum designs for the 384-bar braced barrel vault

Sizing variables	**IBB-BC** [39]	**ECBO** [40]	**IWSA** [41]	**EICA** [42]	**RAO – 1** [43]	**SCA**	**MSCA**
A1	ST 1 1/2	ST 1/2	EST 1/2	ST 1/2	EST 2	ST 3	ST 1 1/2
A2	EST 1 1/2	ST 2 1/2	EST 2	EST 3	ST 1 1/4	ST 4	EST 2
A3	EST 2	EST 2	ST 2 1/2	EST 3	ST 3	EST 2	EST 1 1/2
A4	ST 1 1/2	ST 1 1/2	ST 1 1/4	EST 1 1/2	ST 1 1/4	EST 1 1/4	ST 1 1/4
A5	EST 5	EST 4	EST 4	EST 3 1/2	EST 1 1/4	ST 2 1/2	DEST 3
A6	ST 1 1/2	ST 1 1/2	ST 1 1/4	EST 2 1/2	EST 1 1/4	EST 1 1/4	ST 1 1/2
A7	EST 8	ST 12	ST 12	EST 10	ST 12	ST 12	EST 8
A8	DEST 5	ST 10	DEST 5	ST 10	DEST 6	ST 12	DEST 5
A9	ST 12	ST 12	DEST 6	EST 10	DEST 6	EST 8	EST 10
A10	DEST 6	DEST 8	EST 10	EST 12	EST 8	EST 8	DEST 6
A11	DEST 5	DEST 5	ST 10	EST 8	DEST 5	DEST 5	DEST 5
A12	ST 12	EST 8	EST 8	DEST 5	DEST 5	EST 12	ST 12
A13	EST 5	ST 6	DEST 3	DEST 2 1/2	EST 3	ST 10	DEST 3
A14	EST 4	EST 3 1/2	EST 3 1/2	DEST 2	DEST 8	EST 4	DEST 2 1/2
A15	DEST 2	ST 2 1/2	ST 2 1/2	ST 1/2	EST 2 1/2	EST 3	ST 2 1/2
A16	DEST 3	ST 5	ST 4	ST 4	DEST 2 1/2	ST 8	EST 5
A17	EST 5	EST 4	EST 5	DEST 2	EST 3 1/2	EST 5	DEST 3
A18	ST 1 1/2	EST 1 1/2	EST 1 1/2	EST 4	ST 1 1/4	DEST 5	EST 2
A19	ST 1 1/2	ST 1 1/4	ST 1 1/4	ST 4	ST 1 1/2	ST 8	ST 1 1/4
A20	ST 1 1/2	EST 1 1/2	EST 1 1/2	ST 3/4	EST 1 1/2	ST 4	EST 1 1/2
A21	ST 1 1/2	EST 1 1/2	EST 2	ST 1/2	ST 1 1/2	DEST 3	EST 1 1/2
A22	ST 1 1/2	ST 1 1/4	ST 1 1/4	ST 1 1/2	ST 2 1/2	ST 8	ST 1 1/2

A23	ST 1 1/2	EST 1 1/2	EST 1 ½	ST 2	EST 1 1/2	EST 5	EST 1 1/2
A24	EST 3	EST 2 1/2	DEST 2	DEST 2	ST 2 1/2	DEST 2	ST 3
A25	ST 2 1/2	ST 2 1/2	EST 1 1/2	ST 2 1/2	ST 2 1/2	EST 5	EST 2
A26	ST 2 1/2	ST 2 1/2	ST 2 1/2	ST 2 1/2	ST 1 1/4	ST 4	ST 3
A27	ST 3 1/2	DEST 2	DEST 2	ST 2 1/2	EST 2	ST 8	ST 3
A28	EST 1 1/2	EST 1 1/2	EST 1 1/2	EST 1 1/2	EST 1 1/2	EST 4	EST 1 1/2
A29	EST 2	ST 2 1/2	ST 2 1/2	ST 2	ST 1 1/2	DEST 4	EST 2
A30	EST 1 1/2	EST 1 1/2	EST 2	DEST 2	ST 1 1/2	EST 2	EST 2
A31	EST 1 1/2	EST 1 1/2	EST 2	ST 1 1/2	ST 1 1/2	EST 1 1/2	EST 1 1/2
Best weight (lb)	61,972	62,486.02	61,564.72	61,821.146	61,473.7	95,281.52	**61,421.95**
Mean	62,196	65,785	63,771	65,356.699	N/A	107,030.95	63,540.85
SD	N/A	3,386	1,495	20,84.912	N/A	8,423.17	1,403.77
$NFE_{\max}(NFE)$	50,000	15,980	20,000	7,800	20,000	40,000	40,000 (32,730)

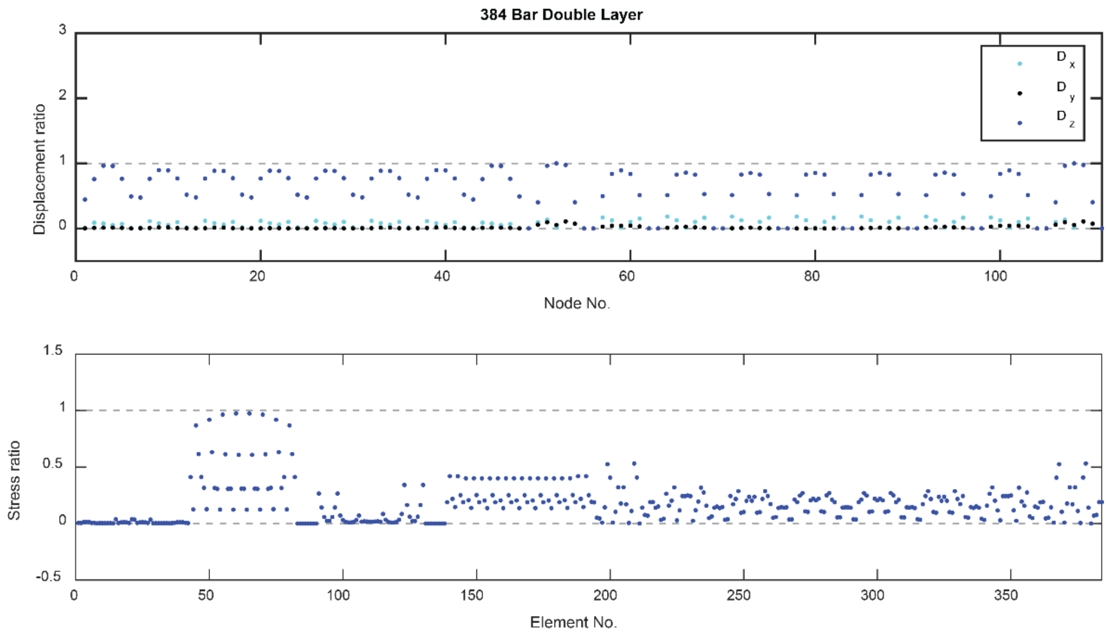

Figure 19. Displacement and Stress ratio of the 384-bar double layer barrel vault (MSCA)

5.2.1 The 582-bar tower

The 582-bar tower is chosen as the last practical design, as depicted in Fig. 20. This structure has been modeled and analyzed with the following characteristics. The elasticity modulus is 204 MPa, and the yield stress is set at 253.1 MPa. The tower is engineered to resist a single load case comprising lateral loads of 5 kN in the X and Y directions, with each node subjected to a downward load of 30 kN. Nodal displacements are limited to 0.08 m in each direction. Stress limits are enforced based on Eq. (7) and Eq. (8). Furthermore, the limitations on the maximum slenderness ratio for tension and compression members are imposed on the structure, set at 300 and 200, respectively. The W-shape profiles are chosen from Table 20.

In this example, the modified version of the sine-cosine algorithm (MSCA), as shown in Table 21, obtained a value of 19.9229 m^3, which outperforms the original version (SCA) and other algorithms in the literature. Furthermore, the algorithm not only outperformed other methods by obtaining the best solution but also accomplished this feat with just 6000 evaluations of the objective function. The objective function evaluation for EVPS is 21000, and for BB-BC, it is 12500, with the corresponding best results being 21.33374 m^3 and 22.371 m^3, respectively.

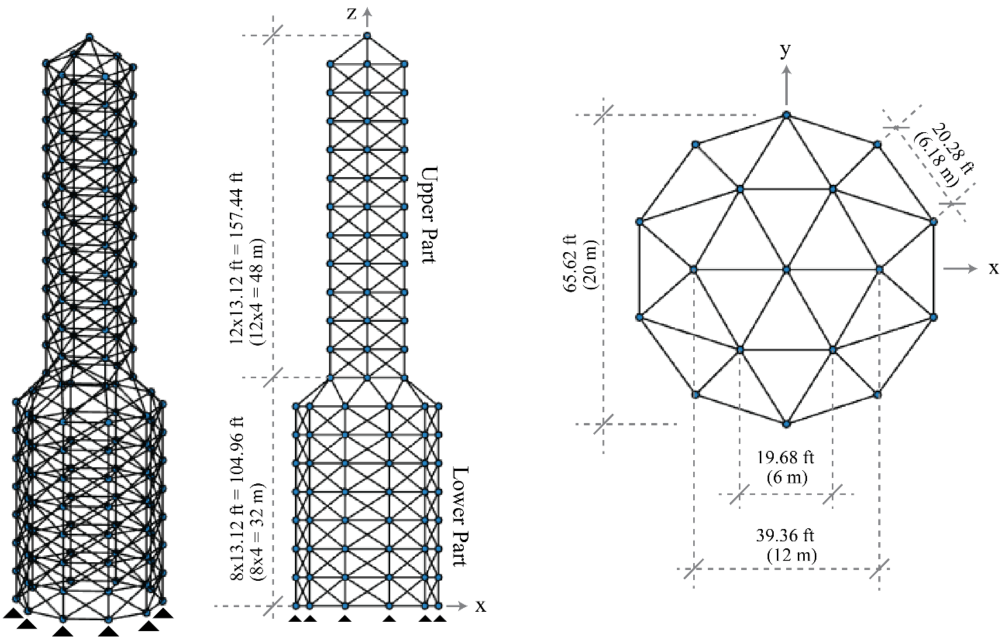

Figure 20. The 582-bar tower

Table 20: Profile list from ASIC code

No.	Profile	No.	Profile	No.	Profile	No.	Profile	No.	Profile	No.	Profile	No.	Profile
1	W8x21	21	W14x48	41	W12x87	61	W33x118	81	W24x176	101	W24x176	121	W30x292
2	W10x22	22	W10x49	42	W10x88	62	W18x119	82	W14x176	102	W14x176	122	W40x297
3	W8x24	23	W12x50	43	W16x89	63	W14x120	83	W27x178	103	W27x178	123	W36x300
4	W6x25	24	W12x53	44	W14x90	64	W21x122	84	W21x182	104	W21x182	124	W14x311
5	W12x26	25	W10x54	45	W21x93	65	W24x131	85	W12x190	105	W12x190	125	W33x318
6	W8x28	26	W12x58	46	W27x94	66	W14x132	86	W30x191	106	W30x191	126	W30x326
7	W12x30	27	W10x60	47	W12x96	67	W12x136	87	W24x192	107	W24x192	127	W36x328
8	W14x30	28	W14x61	48	W18x97	68	W14x145	88	W14x193	108	W14x193	128	W44x335

9	W8x31	29	W21x62	49	W14x99	69	W27x146	89	W27x194	109	W27x194	129	W14x342
10	W10x33	30	W12x65	50	W16x100	70	W24x146	90	W40x199	110	W40x199	130	W33x354
11	W14x34	31	W16x67	51	W10x100	71	W21x147	91	W33x201	111	W33x201	131	W36x359
12	W8x35	32	W10x68	52	W21x101	72	W36x150	92	W30x211	112	W30x211	132	W14x370
13	W16x36	33	W12x72	53	W24x104	73	W12x152	93	W14x211	113	W14x211	133	W14x398
14	W14x38	34	W14x74	54	W12x106	74	W18x158	94	W40x215	114	W40x215	134	W14x426
15	W10x39	35	W18x76	55	W14x109	75	W14x159	95	W27x217	115	W27x217	135	W14x455
16	W8x40	36	W10x77	56	W21x111	76	W27x161	96	W33x221	116	W33x221	136	W14x500
17	W12x40	37	W12x79	57	W10x112	77	W24x162	97	W24x229	117	W24x229	137	W14x550
18	W14x43	38	W14x82	58	W27x114	78	W12x170	98	W36x230	118	W36x230	138	W14x605
19	W12x45	39	W27x84	59	W30x116	79	W30x173	99	W44x230	119	W44x230	139	W14x665
20	W10x45	40	W18x86	60	W24x117	80	W40x174	100	W12x230	120	W12x230	140	W14x730

The convergence diagram of Fig. 21 validates MSCA superior performance over SCA. Based on Fig. 22, it is evident that the constraints of the problem are met within their permissible boundaries, and the degree of constraint violation or infeasibility in the optimal solution is zero.

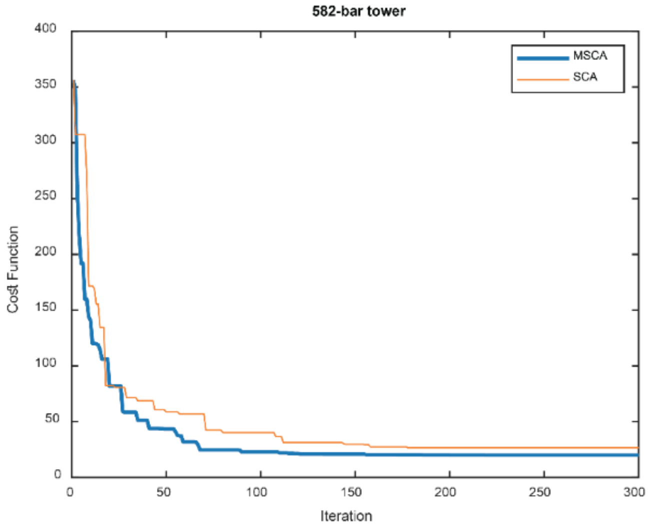

Figure 21. Convergence curve of the 582-bar tower

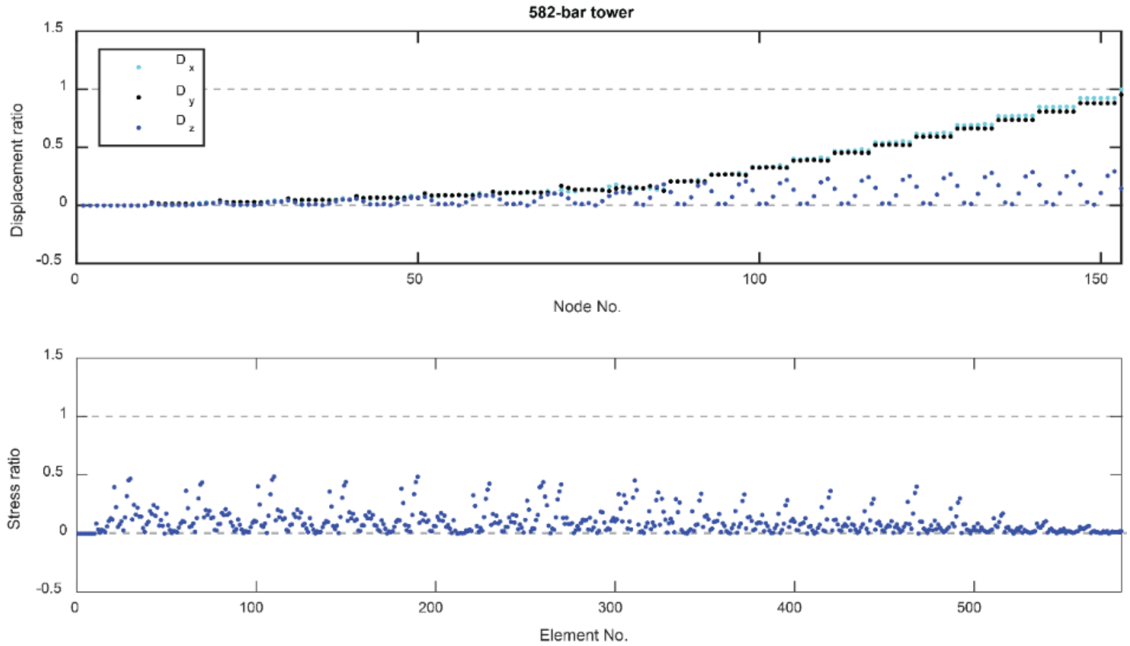

Figure 22. Displacement and Stress ratio of the 582-bar tower (MSCA)

Table 21: Comparison of the optimum designs for the 582-bar tower

Sizing variables	**BB-BC** [44]	**CBO** [45]	**STA** [25]	**EVPS** [46]	**SCA**	**MSCA**
A1	W8x24	W8x21	W8x21	W8x21	W8x21	W8x21
A2	W24x68	W12x79	W10x68	W10x77	W18x97	W18x76
A3	W8x28	W8x28	W8x21	W8x24	W8x21	W8x21
A4	W18x60	W10x60	W10x77	W10x60	W12x72	W14x61
A5	W8x24	W8x24	W8x21	W6x25	W8x21	W8x21
A6	W8x24	W8x21	W8x21	W8x21	W8x21	W8x21
A7	W21x48	W10x68	W10x60	W12x50	W14x38	W10x49
A8	W8x24	W8x24	W8x21	W6x25	W8x21	W8x21
A9	W10x26	W8x21	W8x21	W10x22	W8x21	W8x21
A10	W14x38	W14x48	W14x48	W8x40	W8x21	W12x50
A11	W12x30	W12x26	W8x21	W8x24	W8x24	W8x21
A12	W12x72	W21x62	W14x74	W10x77	W14x211	W10x68
A13	W21x73	W18x76	W16x67	W12x72	W14x90	W12x79
A14	W14x53	W12x53	W12x65	W12x50	W12x26	W10x54
A15	W18x86	W14x61	W12x65	W10x88	W21x101	W18x76
A16	W8x31	W8x40	W8x21	W6x25	W12x50	W8x21
A17	W18x60	W10x54	W12x65	W14x61	W16x89	W12x65
A18	W8x24	W12x26	W8x21	W6x25	W8x21	W8x21

A19	W16x36	W8x21	W8x21	W8x21	W8x21	W8x21
A20	W10x39	W14x43	W10x68	W12x45	W16x36	W8x40
A21	W8x24	W8x24	W8x21	W8x24	W8x21	W8x21
A22	W8x24	W8x21	W8x21	W8x21	W8x21	W8x21
A23	W8x31	W10x22	W10x22	W8x24	W8x21	W8x21
A24	W8x28	W8x24	W8x21	W6x25	W8x21	W8x21
A25	W8x21	W8x21	W12x40	W10x22	W10x33	W8x21
A26	W8x24	W8x21	W6x25	W6x25	W8x21	W12x26
A27	W8x28	W8x24	W10x22	W8x24	W8x21	W8x21
A28	W14x22	W8x21	W8x21	W10x22	W8x21	W8x21
A29	W8x24	W8x21	W8x28	W10x22	W8x21	W8x21
A30	W8x24	W6x25	W10x22	W8x24	W8x21	W8x21
A31	W14x22	W10x33	W16x36	W8x21	W8x21	W8x24
A32	W8x24	W8x28	W12x53	W8x24	W14x38	W8x21
Best ($\boldsymbol{m^3}$)	22.371	21.838	21.130	21.33374	23.0520	**19.9229**
Mean	N/A	N/A	23.738	N/A	26.6777	21.2294
SD	N/A	N/A	N/A	N/A	2.5894	1.9623
$NFE_{\max}(NFE)$	12,500	6,400	6,000	21,000	6,000	6,000 (5,185)

6. CONCLUSION

It was declared that MSCA combines the global-best with a scaled position of the current search agent; that relies on the combination of sine-cosine fluctuations as well as on an envelope function. By tuning the parameters of the envelope function the desired balance is provided between the exploration and the exploitation.

Performance of MSCA was evaluated on thirteen (unimodal and multimodal) test functions by fair comparison with SCA, SPO, PSO, FOA, MVO, MPA and LAPO. Wilcoxon's test statistically confirmed superiority of MSCA over the aforementioned methods by 95% confidence; in 85 cases out of 91 ones.

A set of six engineering benchmarks in discrete, continuous and mixed discrete-continuous types and two discrete practical structural problems was then treated to evaluate the proposed method in constrained optimization. According to the numerical simulation, it was found that MSCA can exhibit superior or competitive results with those reported in literature. Fair comparison with nine other meta-heuristics, better declared relative rank of MSCA in optimizing the real word problems. In the design of coupling with bolted rim, MSCA successfully captured the global best as well as the others meanwhile it revealed better mean result than the others. Although both MSCA and MRFO captured the global solution of the welded-beam problem, MSCA was more robust; revealing much better standard deviation and the mean result. In the other constrained problems, MSCA stood on the first rank by obtaining superior results with respect to the other nine algorithms.

As a practical merit, the proposed method is interesting due to its simple operators. Its control parameters are tunable to provide dynamic balance between exploration and exploitation and to reveal competitive performance with other population-based methods. Particularly, MSCA is an affordable enhanced algorithm with respect to the standard SCA for solution of unconstrained functions as well as engineering benchmark and structural problems.

APPENDIX

A.1 Tension-compression spring problem

Minimize $f(x) = (N+2)Dd^2$

Subjected to

$$g_1(x) = 1 - \frac{D^3 N}{71785 d^4} \leq 0$$

$$g_2(x) = \frac{4D^2 - dD}{12566\left(Dd^3 - d^4\right)} + \frac{1}{5108 d^2} - 1 \leq 0$$

$$g_3(x) = 1 - \frac{140.45 d}{D^2 N} \leq 0$$

$$g_4(x) = \frac{D+d}{1.5} - 1 \leq 0$$

where

$0.05 \leq d \leq 2$, $0.25 \leq D \leq 1.3$ and $2 \leq N \leq 15$.

A.2 The speed reducer problem

Minimize $f(x) = 0.7854bm^2\left(3.3333z^2 + 14.9334z - 43.0934\right) - 1.508b\left(d_1^2 + d_2^2\right)$

$+7.4777\left(d_1^3 + d_2^3\right) + 0.7854\left(l_1 d_1^2 + l_2 d_2^2\right)$

Subjected to

$$g_1(x) = \frac{27}{bm^2 z} - 1 \leq 0$$

$$g_2(x) = \frac{397.5}{bm^2 z^2} - 1 \leq 0$$

$$g_3(x) = \frac{1.93 l_1^3}{mz d_1^4} - 1 \leq 0$$

$$g_4(x) = \frac{1.93 l_2^3}{m z d_2^4} - 1 \le 0$$

$$g_5(x) = \frac{\sqrt{\left(\frac{745 l_1}{m z}\right)^2 + 16.9 \times 10^6}}{\left(110 d_1^3\right)} - 1 \le 0$$

$$g_6(x) = \frac{\sqrt{\left(\frac{745 l_2}{m z}\right)^2 + 157.5 \times 10^6}}{\left(85 d_2^3\right)} - 1 \le 0$$

$$g_7(x) = \frac{mz}{40} - 1 \le 0$$

$$g_8(x) = \frac{5m}{b} - 1 \le 0$$

$$g_9(x) = \frac{b}{12m} - 1 \le 0$$

$$g_{10}(x) = \frac{1.5 d_1 + 1.9}{l_1} - 1 \le 0$$

$$g_{11}(x) = \frac{1.1 d_2 + 1.9}{l_2} - 1 \le 0$$

where

$2.6 \le b \le 3.6$, $0.7 \le m \le 0.8$, $17 \le z \le 28$, $7.3 \le l_1 \le 8.3$, $7.8 \le l_2 \le 8.3$, $2.9 \le d_1 \le 3.9$ and $5 \le d_2 \le 5.5$.

A.3 The welded beam problem

Minimize $f(x) = 1.10471 h^2 l + 0.04811 tb(14.0 + l)$

Subjected to

$$g_1(x) = \tau(x) - \tau_{max} \le 0$$

$$g_2(x) = \sigma(x) - \sigma_{max} \le 0$$

$$g_3(x) = h - b \le 0$$

$$g_4(x) = 0.1047 h^2 + 0.04811 tb(14 + l) - 5.0 \le 0$$

$$g_5(x) = 0.125 - h \le 0$$

$$g_6(x) = \delta(x) - \delta_{max} \le 0$$
$$g_7(x) = P - P_c(x) \le 0$$

where

$P = 6000\ lb$, $L = 14\ in$, $E = 30 \times 10^6\ psi$, $G = 12 \times 10^6\ psi$, $\tau_{max} = 13600\ psi$,

$\sigma_{max} = 30000\ psi$, $\delta_{max} = 0.25\ in$, $0.1 \le h, b \le 2$ and $0.1 \le l, t \le 10$.

$$P_c(x) = \frac{4.013E\sqrt{t^2 b^6 f\ \ 36}}{L^2}\left(1 - \frac{t}{2L}\sqrt{\frac{E}{4G}}\right)$$

$$\sigma(x) = \frac{6PL}{bt^2}$$

$$J = 2\left\{\sqrt{2}hl\left[\frac{l^2}{12} + \frac{(h+t)^2}{4}\right]\right\}$$

$$\delta(x) = \frac{4PL^3}{Et^3 b}$$

$$M = P\left(L + \frac{l}{2}\right)$$

$$R = \sqrt{\frac{l^2}{4} + \frac{(h+t)^2}{4}}$$

$$\tau'' = \frac{MR}{J}$$

$$\tau' = \frac{P}{\sqrt{2}hl}$$

$$\tau = \sqrt{(\tau')^2 + (\tau'')^2 + 2\tau'\tau''\frac{l}{2R}}$$

A.4 The rolling element bearing problem

Maximize $$f(x) = \begin{cases} C_d = -f_c Z^{\frac{2}{3}} D_b^{1.8} & if\ D_b \le 25.4mm \\ C_d = -3.647 f_c Z^{\frac{2}{3}} D_b^{1.4} & if\ D_b > 25.4mm \end{cases}$$

Subjected to

$$g_1\left(x\right)=Z-1-\frac{\varnothing_0}{2sin^{-1}\left(\dfrac{D_b}{D_m}\right)}\le 0$$

$$g_2\left(x\right)=K_{D_{min}}\left(D-d\right)-2D_b\le 0$$

$$g_3\left(x\right)=2D_b-K_{D_{max}}\left(D-d\right)\le 0$$

$$g_4\left(x\right)=D_b-\zeta B_\omega\le 0$$

$$g_5\left(x\right)=0.5\left(D+d\right)-D_m\le 0$$

$$g_6\left(x\right)=D_m-\left(0.5+e\right)\left(D+d\right)\le 0$$

$$g_7\left(x\right)=\varepsilon D_b-0.5\left(D-D_m-D_b\right)\le 0$$

$$g_8\left(x\right)=0.515-f_i\le 0$$

$$g_9\left(x\right)=0.515-f_o\le 0$$

where

$\alpha=0$, $\gamma=\dfrac{D_b\cos\alpha}{D_m}$, $D=160$, $d=90$, $B_\omega=30$, $T=D-d-2D_b$, $0.5\left(D+d\right)\le D_m\le 0.6\left(D+d\right)$, $0.15\left(D-d\right)\le D_b\le 0.45\left(D-d\right)$, $4\le Z\le 50$, $0.515\le f_i\le 0.6$, $0.515\le f_o\le 0.6$, $0.4\le K_{D_{min}}\le 0.5$, $0.6\le K_{D_{max}}\le 0.7$, $0.3\le\varepsilon\le 0.4$, $0.02\le e\le 0.1$ and $0.6\le\zeta\le 0.85$.

$$f_c=37.91\left[1+\left\{1.04\left(\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i\left(2f_o-1\right)}{f_o\left(2f_i-1\right)}\right)^{0.41}\right)\right\}^{10/3}\right]^{-0.3}\left[\frac{\gamma^{0.3}\left(1-\gamma\right)^{1.39}}{\left(1+\gamma\right)^{1/3}}\right]\left[\left(\frac{2f_i}{2f_i-1}\right)^{0.41}\right]$$

$$\varnothing_0=2\pi-2cos^{-1}\left(\frac{\left\{\left(D-d\right)/2-3\left(T/4\right)\right\}^2+\left\{D/2-T/4-D_b\right\}^2-\left\{d/2+T/4\right\}^2}{2\left\{\left(D-d\right)/2-3(T/4\right\}\left\{D/2-T/4-D_b\right\}}\right)$$

A.5 The coupling with bolted rim problem

Table A.5.1 The Discrete list for selection of the bolts

d	$d_e=\varnothing_1\left(d\right)$	$d_2=\varnothing_2\left(d\right)$	$p=\varnothing_3\left(d\right)$	$b_m=\varnothing_4\left(d\right)$	$s_m=\varnothing_5\left(d\right)$
6	5.062	5.350	1.00	7.50	14.50
8	6.827	7.188	1.25	9.50	18.50

10	8.593	9.026	1.50	12.50	23.50
12	10.358	10.863	1.75	13.50	26.50
14	12.124	12.701	2.00	15.50	29.50
16	14.124	14.701	2.00	17.00	32.00
20	17.655	18.376	2.50	21.00	40.00
24	21.185	22.051	3.00	25.00	48.00

Minimize $f(x)=\beta_1\left(\frac{N}{N_m}\right)+\beta_2\left(\frac{R_b+\varnothing_4(d)+c}{R_M}\right)+\beta_3\left(\frac{M}{M_T}\right)$

Subjected to

$$g_1(x)=\frac{\alpha M}{NR_bK(d)}-1\leq 0$$

$$g_2(x)=1-\frac{2\pi R_b}{\varnothing_5(d)N}\leq 0$$

$$g_3(x)=1-\frac{R_b}{\varnothing_4(d)}-R_M\leq 0$$

$$g_4(x)=N-N_{max}\leq 0$$

$$g_5(x)=R_b-R_{Max}\leq 0$$

$$g_6(x)=N_M-N\leq 0$$

$$g_7(x)=R_M-R_b\leq 0$$

$$g_8(x)=M-M_{max}\leq 0$$

$$g_9(x)=M_T-M\leq 0$$

$$g_{10}(x)=d-24\leq 0$$

$$g_{11}(x)=6-d\leq 0$$

where

$\alpha=1.5, R_e=627\,MPa$, $N_M=8$, $N_{max}=100$, $R_M=50\,mm$, $R_{max}=1000\,mm$, $c=5$, $\beta_1=\beta_2=\beta_3=1$, $M_T=40$, $M_{max}=1000$, $f_m=0.15$, $f_1=0.15$, $8\leq N\leq 100$, $50\leq R_B\leq 100$ and $40\leq M\leq 100$.

$$K(d)=\frac{0.9 f_m R_e \pi \varnothing_1^2(d)}{4\sqrt{1+3\left(\frac{0.16\varnothing_3(d)+0.583\varnothing_2(d)f_1}{\varnothing_1(d)}\right)^2}}$$

A.6 The spur gear problem

Minimize $f(x) = \frac{\pi\rho}{4000} \times \left[bm^2 Z_1^2 (1+u^2) - (D_i^2 - d_0^2)(l - b_\omega) - nd_p^2 b_\omega - b(d_1^2 + d_2^2) \right]$

Subjected to

$$g_1(x) = b_1 - \left(\frac{S_n C_s K_r K_{ms} bJm}{k_v K_o k_m} \right) \le 0$$

$$g_2(x) = b_1 - \frac{S_{fe}^2 C_l^2 C_r^2 bD_1 I}{C_p^2 K_v K_o K_m} \le 0$$

$$g_3(x) = 1 + D_2 - \frac{\sin(\varnothing)^2 D_1 (2D_2 + D_1)}{4m} \le 0$$

$$g_4(x) = 8 - \frac{b}{m} \le 0$$

$$g_5(x) = \frac{b}{m} - 16 \le 0$$

$$g_6(x) = b_3 - d_1^3 \le 0$$

$$g_7(x) = b_4 - d_2^3 \le 0$$

$$g_8(x) = \frac{(1+u)mZ_1}{2} - 250 \le 0$$

where

$\rho = 8$, $l = b$, $u = 4$, $l_\omega = 2.5 \times m$, $b_\omega = 3.5 \times m$, $n = 6$, $P = 7.5$, $K_r = 0.814$, $K_{ms} = 1.4$, $K_o = 1$, $K_m = 1.3$, $C_l = 1$, $C_r = 1$, $\varnothing = 25$, $C_P = 191$, $\tau = 19.62$, $Dr = m \times (aZ_1 - 2.5)$, $D_i = D_r - 2l_\omega$, $d_0 = d_2 + 25$, $d_P = 0.25(D_i - d_0)$, $D_1 = mZ_1$, $N_1 = 1500$, $v = \pi D_1 N_1 f\ \ 60000$, $b_1 = 1000P f\ \ v$, $S_n = 1.7236H$, $C_s = -0.0007548H + 0.8899$, $K_v = \left(78 + \sqrt{196.85v}\right)/78$, $S_{f_e} = 2.8H - 69$, $I = a \times \sin(\varnothing) \times \cos(\varnothing)/2(a+1)$, $J = 1.766E - 6z_1^3 - 0.0002996z_1^2 + 0.01772z_1 + 0.1608$, $D_2 = amZ_1$, $b_3 = 4.97E6 \times P / N_1\tau$, $N_2 = N_1 f\ \ a$, $b_4 = 4.97E6 \times P / N_2\tau$, $b \in [10, 35mm]$, $d_1 \in [10, 30mm]$, $d_2 \in [10, 40mm]$, $Z_1 \in [18, 25]$, $m \in \{1, 1.25, 1.5, 2, 2.75, 3, 3.5, 4\}$ and $H \in [200, 400]$.

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