BASIS PURSUIT BASED GENETIC ALGORITHM FOR DAMAGE IDENTIFICATION

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ABSTRACT

In damage detection the number of elements is generally more than the number of measured frequencies. Consequently, the corresponding damage detection equation is undetermined and thus has infinite solutions. Since in the damaged structures most of their elements remain healthy, the sparsest solution for the damage detection equation is mostly the actual damage. In the proposed method, the damage equation is first linearized in various ways using random finite difference increments. The sparsest solutions for created linear system of equations are derived using basis pursuit. These solutions are considered as the first population for a continuous genetic algorithm to obtain the damage solution. For investigation of the proposed method three case studies are considered. Simulation results confirm the efficiency of the proposed method compared to those found in the literature.

1. INTRODUCTION

Identifying structural damage by using nondestructive test data has been investigated by many researchers during the past two decades. Vibration testing is the most widely used method for identifying the parameters of structures. Most existing parametric methods for dynamic identification make use of frequencies and mode shapes and are based on finite element (FE) model updating [1-3]. Mostly, the damage parameters are identified by equalizing the

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responses of analytical model to the measured responses. In this way, the problem is modelled as a nonlinear system of equations in which the damage parameters should be determined. Some common methods for solving the system of equations are pseudo-inverse, least square, non-negative least square and optimization methods. Genetic algorithm (GA), as an optimization method, has been frequently employed to solve the corresponding system of equations for damage detection [2].

Au et al. [3] by expansion of incomplete mode shapes obtained energy quotient difference to find the most potentially damaged elements. Then, they detected the damages of the limited elements by micro genetic algorithm. For large scale structures, they proposed a two level optimization method in which the subset of the damaged elements was searched.

Gue and Li [4] proposed a two-stage method to determine the location and extent of multiple structural damages. At the first stage, the damaged sites were localized using the evidence theory by frequencies and mode shapes data. At the second one, a micro search genetic algorithm was employed to detect the true sites and extents. He and Hwang [5] first reduced the design variable of damage detection problem by a grey relation analysis and furthermore, introduced an improved real coded genetic algorithm with a new mutation operator which merges the merit of simulated annealing. Naseralavi et al. [6] improved genetic algorithm to detect damages and cracks. They embedded two additional operators in GA after the usual genetic operators to restrict the design variables. The first additional operator improved the genetic algorithm using the sensitivity analysis for each individual. The second one eliminated the small damage variables after each several generations.

For solving the sparse solution of an undetermined linear system of equations Orthogonal Matching Pursuit (OMP) and Basis Pursuit (BP) are well known [7]. Since the system of equations for damage detection is slightly nonlinear only, OMP and BP (themselves or their ideas) have been employed. Meruane and Heylen [8] adopted the above concept of BP through adding the summation of damage variables as a penalty function to the conventional objective function. Friswell and co-workers frequently employed OMP for damage identification with a new name as forward selection. They confirmed the effectiveness of OMP [9-11] for damage detection. To select the number of damage parameters in OMP, Friswell et al. employed Efroymson's criterion in Ref. [10]. In that work, OMP is extended to the case with multiple measurement sets using angles between subspaces. Friswell et al. [11] also extended OMP for parameter groups. OMP [12] was applied to identify the damages of bearings effectively in Ref. [12]. Yang et al. [13] presented a procedure which combined BP and a feed forward neural network classifier to detect the fault of rolling element bearings. Also, the results of BP and MP were compared using vibration analysis. The comparison demonstrated that basis pursuit feature-based fault diagnosis was more accurate than matching pursuit feature-based fault diagnosis for detecting the faults. Practically, the measured responses are noisy. The Basis Pursuit De-Noising model (BPDN) is commonly used in noisy linear system of equations [14-16]. Many researches study the application of BPDN for compressed sensing [17-19].

In this paper, a new method is presented to detect the structural damages. First, the sensitivity matrices of structural responses with respect to elemental damages are evaluated using the finite difference method with various finite difference increments. Then, various systems of equations are formed for the structure and solved by BP. The obtained solutions

are considered as the first population for a continuous genetic algorithm. In an optimization process the genetic algorithm improves the results of the first stage to the true damage. The efficiency of the proposed method is compared with the algorithms in the literature.

The organization of this paper is as follows: The linearization of damage detection is discussed in section 2. The Basis Pursuit is described in section 3. Genetic algorithm is explained is section 4. The proposed algorithm and three illustrative case studies are presented in sections 5 and 6, respectively. Finally, the conclusion is given in section 7.

2. LINEARIZATION OF DAMAGE DETECTION

Damage detection problems can be formed as a set of equations. To solve the equations, damage variables should be found in a way to best equalize the analytical and measured responses of the structure. Mathematically, the set of equations can be expressed as:

$$\mathbf{R}_d = \mathbf{R}(\mathbf{X}) \tag{1}$$

where $\mathbf{X} = \left(x_1, x_2, \dots x_n\right)^{\mathrm{T}}$ is the vector of damage variables and n is the number of structural elements. x_i is the ratio of the stiffness lost in the damaged element to the stiffness of intact state for the ith element that is called damage ratio. $\mathbf{R}_d = \left(r_{d_1}, r_{d_2}, \dots r_{d_m}\right)^{\mathrm{T}}$ is the vector of m structural responses of measured damage structure. The vector of m responses of analytical model is denoted by $\mathbf{R}(\mathbf{X}) = \left(r_1(\mathbf{X}), r_2(\mathbf{X}), \dots r_m(\mathbf{X})\right)^{\mathrm{T}}$.

Practically for modal data, the number of structural responses, m, is less than the number of elements, n, and thus, the problem is undetermined. Hence, the solution of this problem fails to be unique. To find the true unique solution, a constraint should be inserted to the problem. Since in damage cases most of the structural elements are still remained healthy (a few elements are damaged), the true damage solution is spars (it has a few nonzero entries in comparison to its dimension). This constraint helps us to find the desirable solution of damage detection problem. Thus, the main object is to find the sparsest solution of Eq. (1).

Since R(X) is slightly nonlinear, it is expected that the sparsest solution of $\mathbf{R}_d = \mathbf{R}(\mathbf{X})$ be close to the sparsest solution of its linear version. The first order approximation of Eq. (1) is as follows:

$$\mathbf{R}_d = \mathbf{R}(\mathbf{X}) = \mathbf{R}_h + \frac{\partial \mathbf{R}}{\partial \mathbf{X}} \mathbf{X} + \dots \Rightarrow \mathbf{R}_d - \mathbf{R}_h = \Delta \mathbf{R} \cong \mathbf{SX}, \tag{2}$$

in which, $\mathbf{R}_h$, $\mathbf{X}$ and $\mathbf{S}$ are the response vector of the healthy structure, damage vector and sensitivity matrix of structural responses, respectively. In the next section we will see how to find the sparsest solution of a linear system of equations.

3. BASIS PURSUIT (BP)

Consider the following linear system of equations:

$$\mathbf{A}_{m \times n} \mathbf{X}_{n \times 1} = \mathbf{b}_{m \times 1}, \tag{3}$$

where $\mathbf{X}$ is the vector of unknowns. This linear system of equations stands for $\mathbf{SX} = \Delta \mathbf{R}$. For undetermined problems ($m < n$), Eq. (3) has infinite solutions among them the sparsest one is mostly desired. For a vector such as $\mathbf{X}$, l_p -norm ($p = 0, 1, 2, ...$) are defined as follows [12, 13]:

$$\|\mathbf{X}\|_p = \begin{cases} \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p} , 0 < p < \infty \\ \max_{i=1,...,n} |x_j|, \ p = \infty \end{cases} \tag{4}$$

where $|.|$ denotes the absolute value. Consequently, $\|\mathbf{X}\|_0$ (l_0 -norm) is the number of non-zero entries of $\mathbf{X}$. Mathematically speaking, we can write:

$$\|\mathbf{X}\|_0 = \left\{ 1 \le i \le n : x_i \ne 0 \right\}, \tag{5}$$

$\|\mathbf{X}\|_1$ is the summation of the absolute values for components of $\mathbf{X}$, i.e.

$$\|\mathbf{X}\|_1 = \sum_{i=1}^{n} |x_i|. \tag{6}$$

Also, $\|\mathbf{X}\|_2$ (l_2 -norm) is equal to:

$$\|\mathbf{X}\|_2 = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}, \tag{7}$$

To find the sparsest solution of Eq. (3), l_0 -norm of $\mathbf{X}$ should be minimized. The solution with the smallest l_0 is represented as:

$$\mathbf{X}_0 = \arg \min_{\mathbf{X}:\mathbf{AX}=\mathbf{b}} \|\mathbf{X}\|_0. \tag{8}$$

Unfortunately, l_0 minimization is not a convex optimization problem. In fact, it is generally an NP-hard problem [29]. That is, Eq. (8) requires beyond a polynomial time to be solved through l_0 minimization.

The solution with minimum l_1 is denoted as:

$$\mathbf{X}_1 = \arg\min_{\mathbf{X}:\mathbf{AX}=\mathbf{b}} \|\mathbf{X}\|_1 . \tag{9}$$

It can be shown that the solutions with minimal l_0 and l_1, namely $\mathbf{X}_0$ and $\mathbf{X}_1$, are mostly equal. Thus, Eq. (8) can be solved through l_1 minimization instead of l_0 minimization. l_1 minimization is a convex optimization problem and can be solved in polynomial time. This strategy is called Basis Pursuit (BP) [11]. Equation (9) is an optimization with a linear objective function, $\|\mathbf{X}\|_1$, and linear equality constraint $\mathbf{AX}=\mathbf{b}$. Thus Equation (9) can be solved through a Linear Programming (LP). Generally, simplex method is an efficient tool to solve linear programming problems [14].

Note that computation of the solution with minimum l_2-norm, $\mathbf{X}_2 = \arg\min_{\mathbf{X}:\mathbf{AX}=\mathbf{b}} \|\mathbf{X}\|_2$, is much easier rather than $\mathbf{X}_1$. The solution with minimal $\mathbf{X}_2$ can be achieved by $\mathbf{X} = \mathbf{A}^+\mathbf{b}$ where the superscript "+" represents the Moore-Penrose pseudo inverse. However, unfortunately $\mathbf{X}_2$ is close to $\mathbf{X}_0$ and it is not generally equal to that.

Noisy version of Eq. (3) is expressed as $\mathbf{AX} = \mathbf{b} + \mathbf{z}$, in which $\mathbf{z}$ is the noise vector. It is assumed $\|\mathbf{z}\|_2 \le e$, where e is an upper bound for noisiness. The l_0 minimization problem in the noisy state can be expressed as:

$$\min_{\mathbf{X}} \|\mathbf{X}\|_0 \quad \text{subject to} \quad \|\mathbf{AX} - \mathbf{b}\|_2 \le e, \tag{10}$$

which is similar to Eq. (8) where the equality constraint is relaxed to an inequality constraint.

The noise vector is simulated as $\mathbf{z} = s \times \mathbf{z}'$ where s and $\mathbf{z}'$ are the noise level and the standard white Gaussian noise, respectively. On the other hand, the l_0-norm can be replaced by the l_1-norm for pursuing the strategy of convex relaxation:

$$\min_{x} \|\mathbf{X}\|_1 \quad \text{subject to} \quad \|\mathbf{AX} - \mathbf{b}\|_2 \le e. \tag{11}$$

This strategy is called Basis Pursuit De-Noising (BPDN) [11]. Lagrangian for this minimization problem is written as follows:

$$\min_{x} \quad \frac{1}{2}\|\mathbf{AX} - \mathbf{b}\|_2^2 + l\,\|\mathbf{X}\|_1 . \tag{12}$$

In fact, Eq. (12) can be solved by quadratic programming. The solutions of Eqations. (11) and (12) are the same for an appropriate $l > 0$. The parameter l makes a balance between minimizing error and sparsity. l is set to the value $l_p = s\sqrt{2\log(p)}$ where p is the cardinality of $\mathbf{A}$ [14].

4. GENETIC ALGORITHM

Genetic algorithm is an efficient meta-heuristic optimization tool which mimics the population enhancements during the generations. In the conventional genetic algorithm (discrete version) each solution point is coded as a binary string. But continuous genetic algorithm (CGA) uses directly the variables themselves which is better for optimization problems with continuous variables. Thus, coding and decoding process is not needed in CGA. Generally in GA (or CGA), each individual of the population is called chromosome and each variable is called gene. In genetic algorithm feasible initial population is created randomly at first. The objective function for the individuals of the current population is calculated. Then, it is decided which chromosomes in the current population are fit enough to survive and possibly reproduce offspring in the next generation in selection operator. The individuals with better fitness are copied more than others. Reproduction process usually consists of two main operators: crossover and mutation. Crossover simulates marriage and generation of offspring by combining two individuals as parents. Mutation is an operator which simulates biological genetic mutation by changing some genes randomly. The algorithm is generally stopped based on the maximum number of generations [9].

5. THE PROPOSED ALGORITHM: BP-CGA

Practically, in damage detection via modal data the number of measured structural responses, m, is less than the number of elements, n. Hence, the damage detection problem is undetermined and its solutions make an $n - m$ dimensional subspace in R^n. Usually, the number of damaged elements is a few and most of the elements are healthy. Hence, $\|\mathbf{X}\|_0 << n$ and thus it is implied that the solution with highest sparsity is the best answer to the problem. To find the sparsest solution, the l_1-norm is minimized (basis pursuit).

Conventionally, BP is used for linear system of equations while the damage detection problem is a nonlinear system of equations. Linearization error for $\mathbf{R}_d = \mathbf{R(X)}$ can be considered as an unbiased noise. Also, the damage detection data is practically noisy. Therefore, in damage detection we have to deal with a noisy version of $\mathbf{SX} = \Delta \mathbf{R}$. However, practically the linearization error and noisiness amount is insignificant. In other words, BP results in an approximate solution for damage problem which is near enough to the exact solution. If we generated the first population uniformly over the search space, CGA would converge to one of the many solutions of Eq. (1) which is not essentially the true damage solution. However, BP part helps to produce the first generation near the exact solution. Therefore, when CGA begins with such a population, it will converge to the true damage solution.

The method consists of two main parts as follows:
1) Generation of the first population:

For production of each individual in the initial population, first the vector $\mathbf{e}^j = (e_1^j, e_2^j, ..., e_n^j)$ is produced in which e_i^j ;$i = 1, 2, ..., n$ are random numbers in the interval $[0,1]$. The entries of this vector are used as the finite difference increments for

creating the columns of sensitivity matrix of the structure as follows:

$$\mathbf{S}_i^j = \frac{\mathbf{R}_{d_i}^j - \mathbf{R}_h}{e_i^j}, \tag{13}$$

where $\mathbf{R}_{d_i}^j$ is the structural responses for the jth individual when the damage ratio of the ith element is equal to e_i^j and the other elements are considered healthy, i.e.

$$\mathbf{R}_{d_i}^j = (0,0,..., \overset{ith\ entry}{\overset{\uparrow}{e_i^j}}, 0,..,0).$$ $\mathbf{S}_i^j$ is the ith column of the jth sensitivity matrix corresponding to the jth individual. Therefore, sensitivity matrices of the individuals are considered as $\mathbf{S}^j = \left[\mathbf{S}_1^j, \mathbf{S}_2^j, ..., \mathbf{S}_n^j \right]; \ j = 1, 2, ... N_{ch}$, where N_{ch} is the population size. The corresponding system of equations are $\mathbf{S}^j \mathbf{X}^j = \Delta\mathbf{R}; \ j = 1, 2, ... N_{ch}$. By solving the sparsest solution for these equations through employing BP, the first population of CGA is generated. Using various finite difference increments for linearizing $\mathbf{R}_d = \mathbf{R}(\mathbf{X})$, the nonlinearity problems of $\mathbf{R}(\mathbf{X})$ is overcome.

2) Continuous Genetic Algorithm (CGA):

CGA improves the approximate damage solutions in the first population to the exact one. The objective function for CGA is considered as $\left\| \mathbf{R}_d - \mathbf{R}(\mathbf{X}) \right\|_2$.

Figure 1 shows the flowchart of the proposed algorithm. In this flowchart, the number of generations is denoted by N_g. In the next section, we will see several test examples to show the effectiveness of BP-CGA.
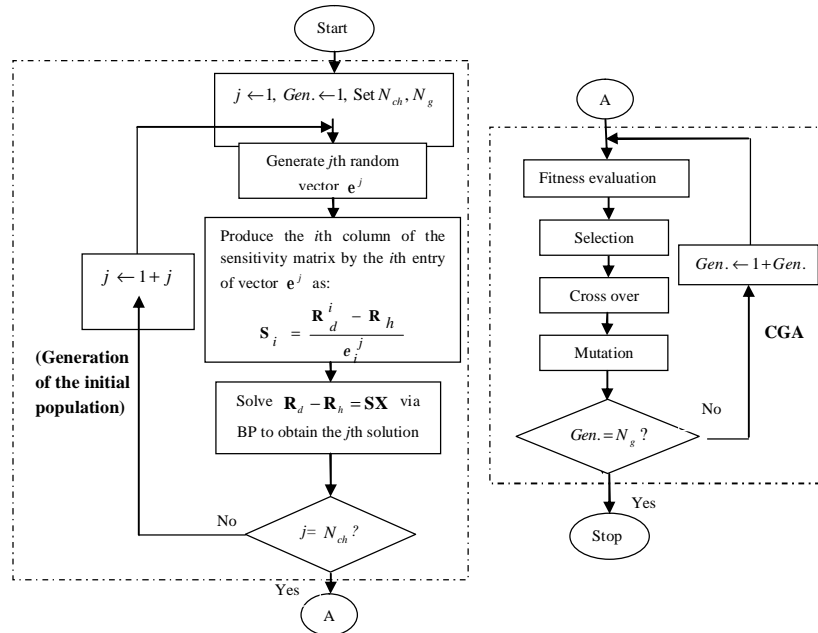

Figure 1. The flowchart of the improved CGA algorithm

6. CASE STUDIES

In this part, the proposed method is verified by three different case studies. Each numerical example is investigated in both cases of noise-free and noisy data. The ith noisy response is simulated by $(1+sg)r_{d_i}$, where s is the noise level and g is a random number in the interval $[-1,\ 1]$. In the following case studies, damage is simulated by reductions in Young's modulus of the damaged elements. To demonstrate the efficiency and accuracy of the proposed method, the results are compared with the results of CGA-SBI-MS algorithm [6]. An index called *error in detection* is defined as $ED = \|\mathbf{X} \text{-} \mathbf{X}_d\|_2$, where $\mathbf{X}_d$ represents the actual damage vector [6]. In the following case studies, for genetic algorithm, the population size, the crossover and mutation probabilities are set to 50, 0.8, and 0.015, respectively.

6.1. A cantilever beam

A fifteen-element cantilever beam is simulated to illustrate the efficiency of the proposed method. This structure has been previously studied by Koh and Dyke [20]. The geometrical and physical properties are as follows: the length of beam is 2.74 m; the elasticity modulus is $2 \times 10^{11} \text{N} / \text{m}^2$; the thickness and width are 0.00635 and 0.0760 m, respectively. The elements are numbered from the fixed end as follows:
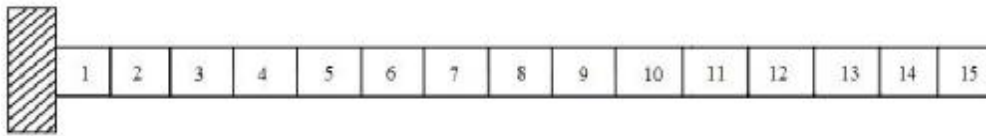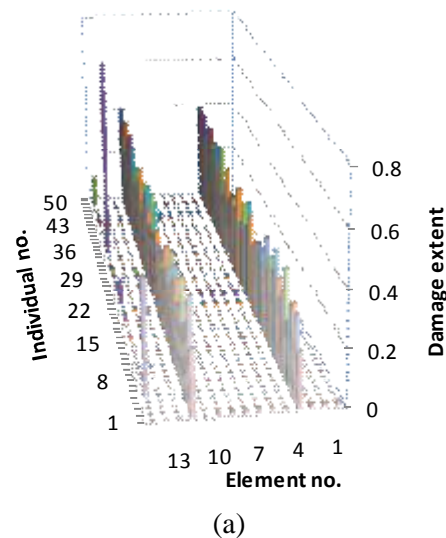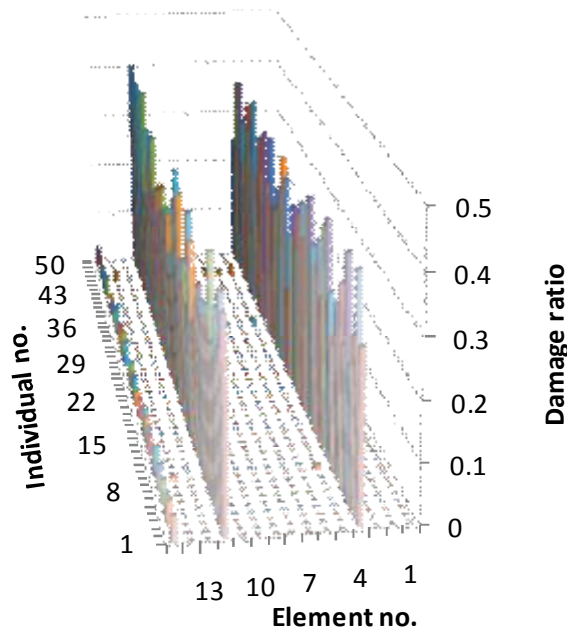

Figure 2. A 15-element cantilever beam

The 4th and 12th elements of the cantilever beam are assumed to be damaged by the extent of 30%. The first five frequencies are used to detect the damaged elements.


(a)

(b)

Figure 3. The first stage results for 15-element cantilever beam: (a) damage identification results without noise (b) damage identification results with 1% noise
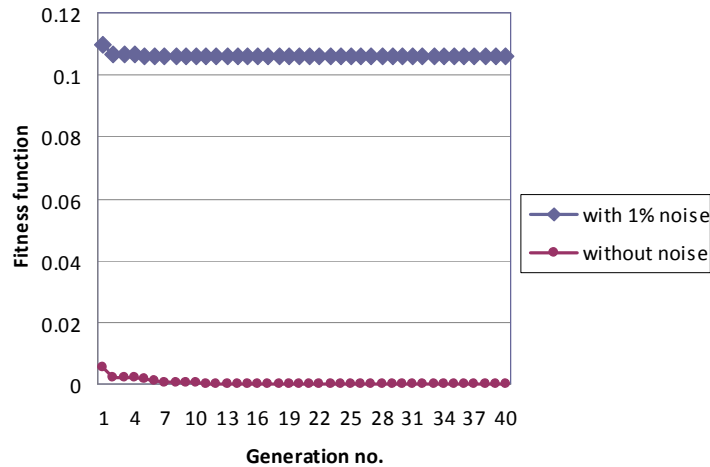
Figure 3 shows the output of BP part for generation of the first population in both noise-free and noisy frequencies. As it is observed, the BP part can successfully generate the first population of CGA near to the true damage. This point is very helpful by quadratic programming converging of CGA algorithm to the actual damage.
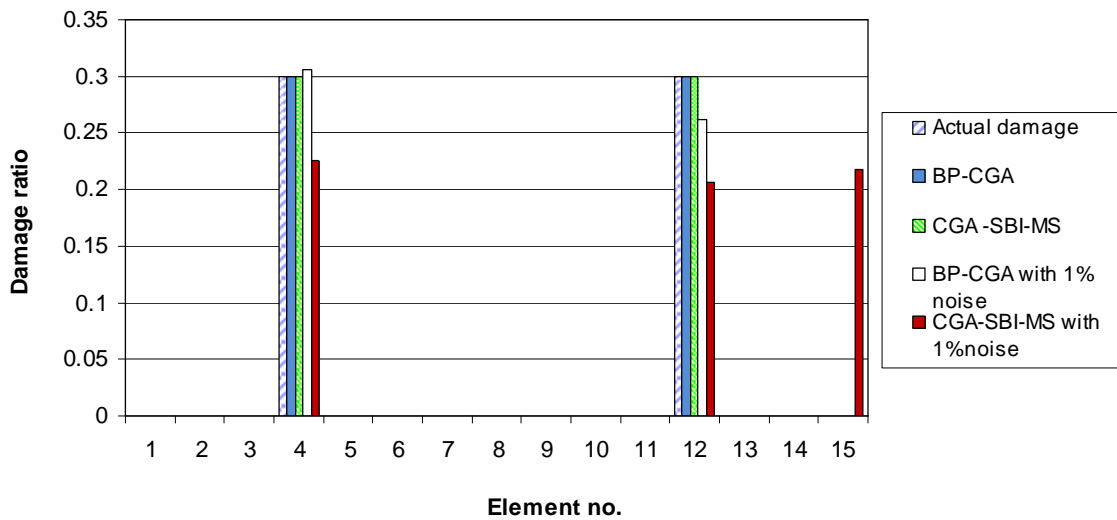
(a)
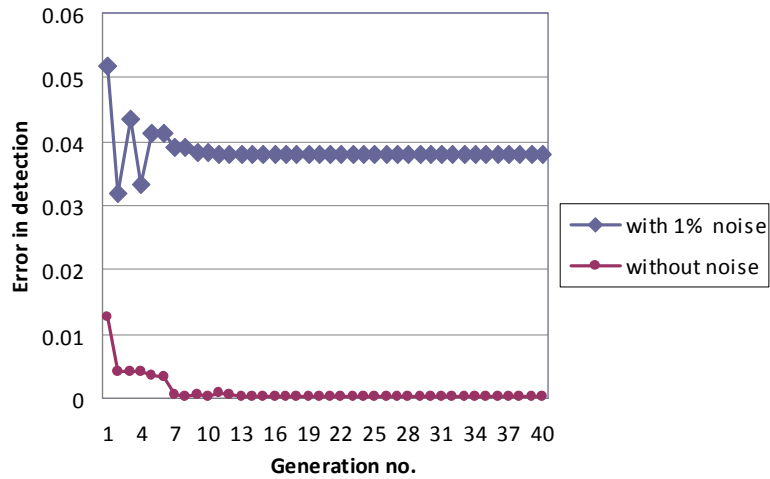
(b)

(c)

Figure 4. Solution results for 15-element cantilever beam without and with 1% noise: (a) damage identification results (b) convergence history of error in detection (c) convergence history of fitness function

The results of CGA part are presented in Figure 4. As it can be seen from Figure 4(a), the damage elements and ratios are approximately detected by BP-CGA. This method can exactly identify the damages similar to CGA-SBI-MS in noise-free cases. But the performance of BP-CGA is better than that of CGA-SBI-MS in noisy data. As it can be seen CGA-SBI-MS wrongly reports element 15 as the damage elements. Figures 4(b) and (c) show the convergence history of errors in detection and fitness function during the optimization process. In order to cancel out the stochastic nature of optimization process, the algorithm is independently run twenty times and the average of results is given in Table 1. By comparing the damage detection results of the proposed method and CGA-SBI-MS, it is revealed that the proposed algorithm has a better performance in noisy data.

Table 1. The average results for twenty times running for 15-element cantilever beam

Element number	Damage ratio	BP-CGA	CGA-SBI-MS
4	0.3	0.3	0.25
5	0	0	0.14
12	0.3	0.28	0.28
15	0	0	0

6.2. A planar truss

A 31-bar planar truss which has been studied by Messina et al. [21] is selected to demonstrate the capability of the proposed algorithm. The structure is shown in Figure 5. The damages are to be detected by using the first 10 frequencies.
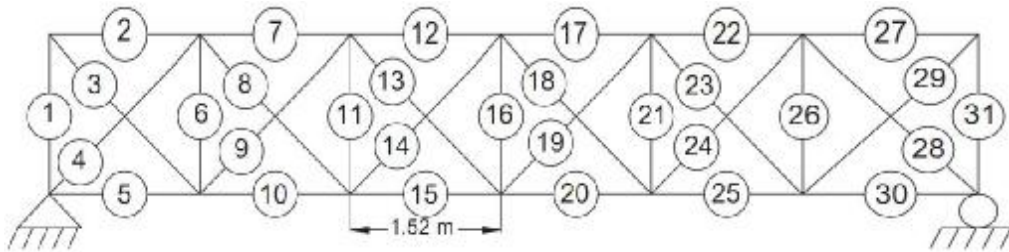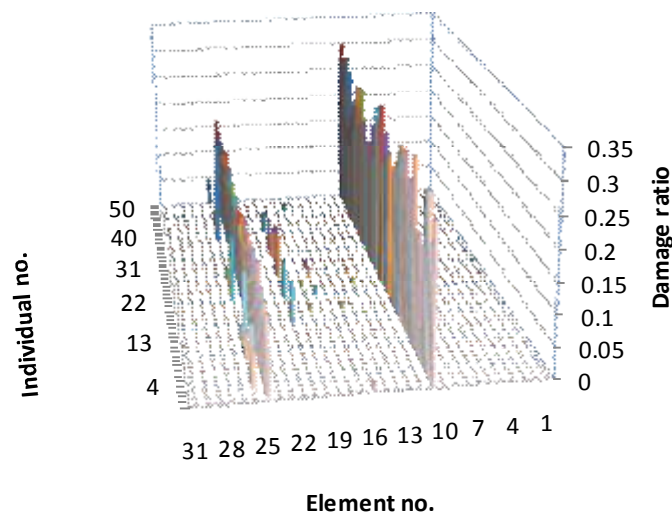

Figure 5. A 31-bar planar truss

The 11th and 25th elements of the planar truss are considered to be damaged by the extent of 25% and 15%, respectively. The output of BP for generation of the first population is shown in Figure 6. As it can be seen in Figure 6, the proposed method generates the first population of CGA in both noise-free and noisy data close to the actual damage.
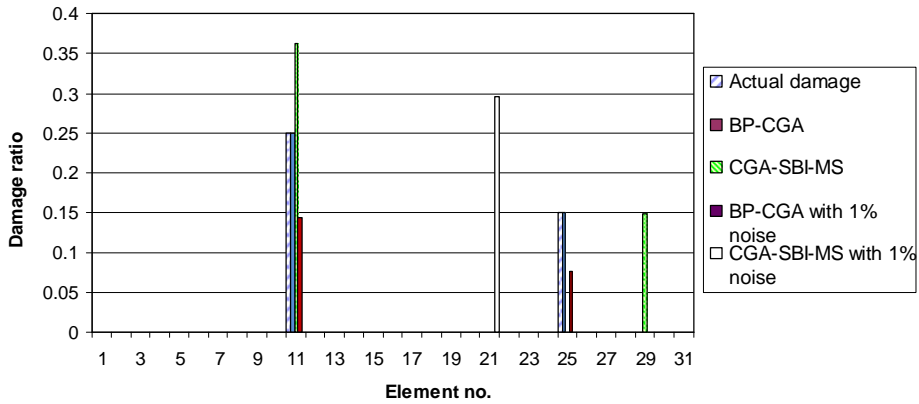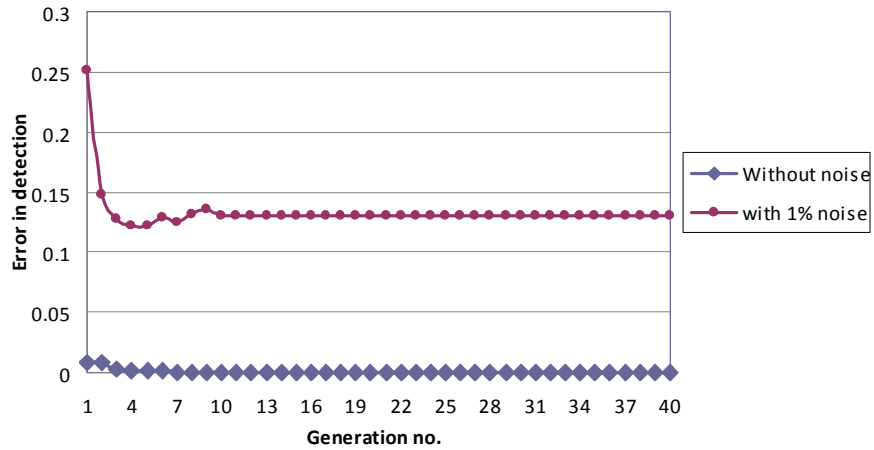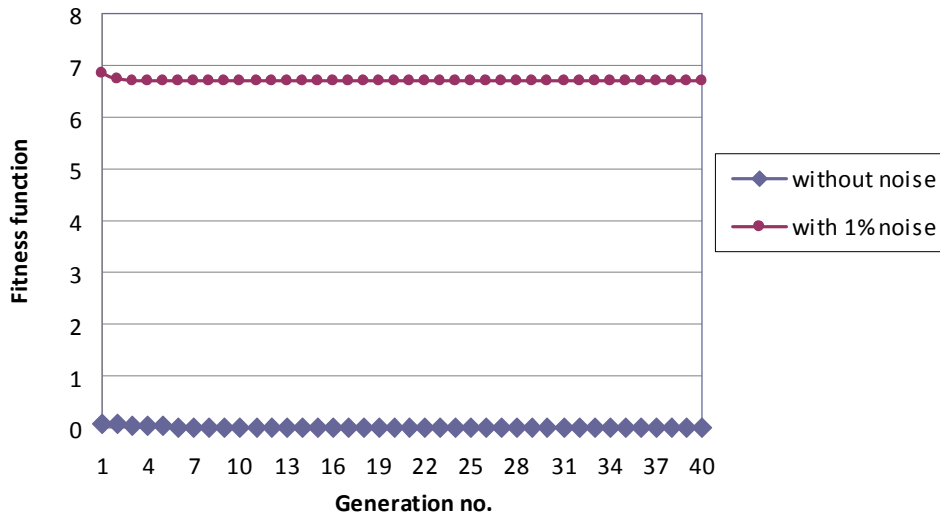

(a)
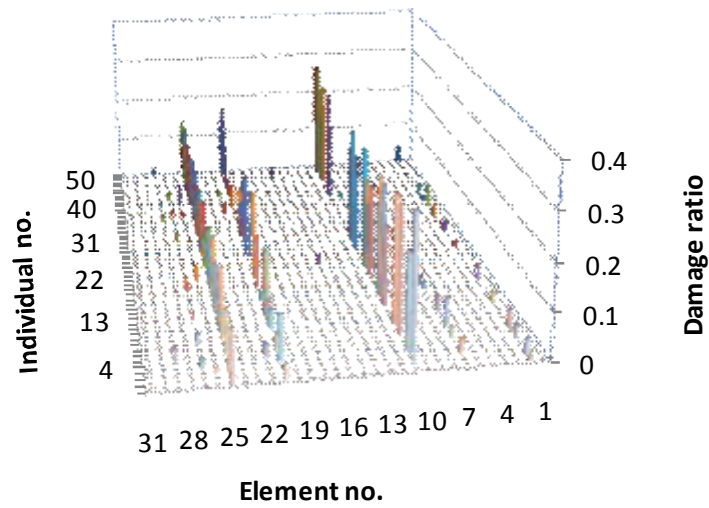
(b)

Figure 6. The first stage results for 31-element planar truss (a) without noise (b) with 1% noise

(a)

(b)

(c)

Figure 7. Solution results for the 31-element planar truss: (a) damage identification results (b) convergence history of error in detection during the process (c) convergence history of fitness function

Figure 7(a) depicts final results for BP-CGA and CGA-SBI-MS for both cases of noise-free and noisy data. As observed the results of both algorithms are completely exact in noise-free data, but the results of BP-CGA is somewhat better in noisy data. The average of BP-CGA results with no noise are compared with CGA-SBI-MS ones in Table 2.

Table 2. The average results for twenty times running of the algorithm for the planar truss

Element number	Damage ratio	BP-CGA	CGA-SBI-MS
11	0.25	0.25	0.31
21	0	0	0.3
25	0.15	0.15	0
29	0	0	0.1

In Table 3, BP-CGA is also compared with MGA+ECBI [22] for 0.15% noise level. BP-CGA is executed ten times to have a fair comparison. The results are presented in the table. Finally the average of the ten outputs is given. As it can be seen, the result of BP-CGA is better than that of MGA+ECBI.

Table 3. The damage ratios of the 31-bar truss using BP-CGA and the MGA with the ECBI considering noise

Sample no.	Element no.																
	1	...	7	...	11	...	20	21	...	25	26	27	28	29	30	31
1			0.012		0.245			0.062		0.136	0.01					
2			0		0.283			0.027		0.156	0					
3			0		0.209			0		0.157	0					
4			0.023		0.256			0		0.122	0					
5			0.017		0.225			0.072		0.136	0					
6			0		0.298			0		0.148	0					
7			0		0.245			0		0.157	0					
8			0.019		0.223			0.032		0.146	0					
9			0		0.273			0		0.148	0					
10			0.015		0.229			0.017		0.133	0					
Average	0	0	0.008	0	0.249	0	0	0.021	0	0.144	0.001	0	0	0	0	0
MGA+ECBI	0	0	0	0	0.285	0	0	0.06	0	0.135	0	0.005	0	0.005	0.015	0
Actual damage	0	0	0	0	0.25	0	0	0	0	0.15	0	0	0	0	0	0

6.3. A prestressed concrete beam

A fifteen-element prestressed concrete beam is investigated to verify the ability of the algorithm. The tendon of the beam is straight passed through the centre of the cross section. Thus the prestress force of the tendon acts as an axial force for the beam (no bending moment). The structure is shown in Figure 8. The geometrical properties are the same as the first case study. Young's modulus, density and prestress force are equal to 30.2 GPa, 2351.4 kg/m^3 and 100 kg, respectively.
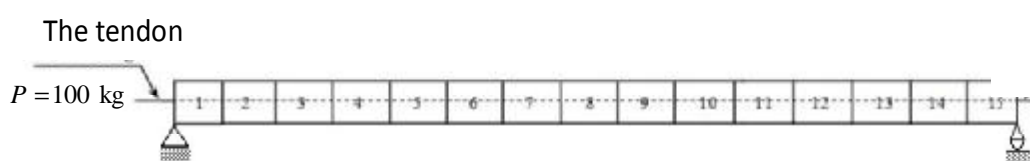

Figure 8. A 15-element prestressed concrete beam

In fact the presence of compression force leads to reductions in natural frequencies of the structure. To show this, natural frequencies of the beam in both cases with and without axial force are given in Table 4. The stiffness matrix of the beam element with an axial force is given in Appendix I. The 4th and 12th elements of the beam are assumed to be damaged by the extent of 30%. The average results of twenty times running BP-CGA are compared with the average results of CGA-SBI-MS algorithm. The results in both cases of no noise and noise level of 1% are given in Table 5.

Table 4. The frequencies of the concrete beam with and without prestress force

	Frq. 1	Frq. 2	Frq. 3	Frq. 4	Frq. 5
No axial force	1.3732	5.4776	12.2688	21.6762	33.6068
With axial force	1.3372	5.4420	12.2335	21.6411	33.5721

Table 5. The average results for twenty times running for 15-element prestressed concrete beam

Element number	Damage ratio	BP-CGA		CGA-SBI-MS	
		No noise	1% noise	No noise	1% noise
4	0.3	0.23	0.19	0.31	0.31
12	0.3	0.35	0.24	0.25	0.16

The results of the BP part of the algorithm for generation of the initial population are shown in Figure 9. Also the final result of the algorithm and convergence histories of fitness function and error in detection during CGA process are depicted in Figure 10.

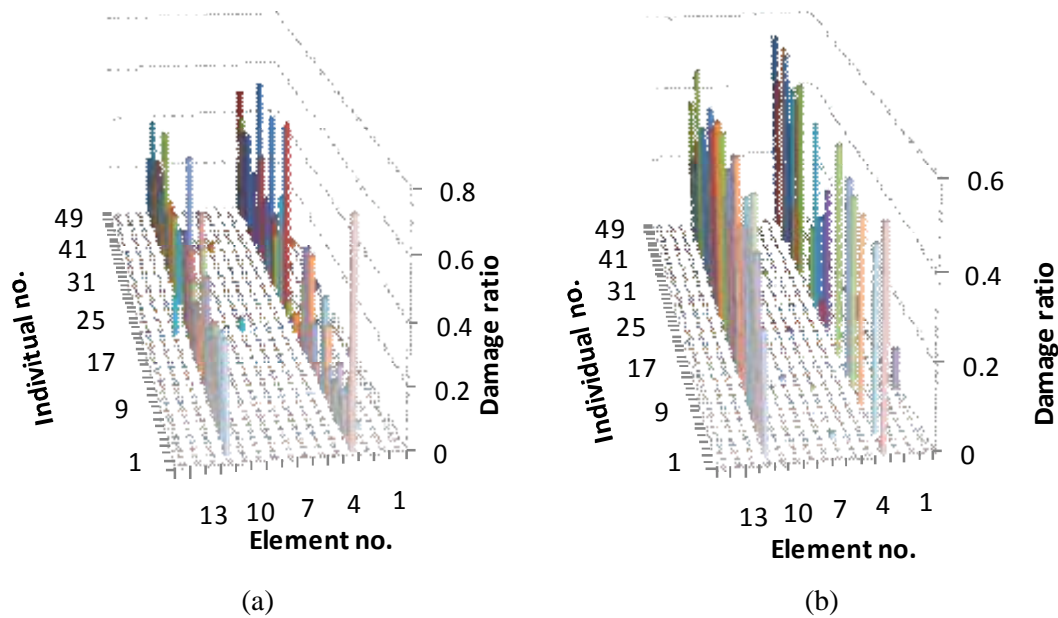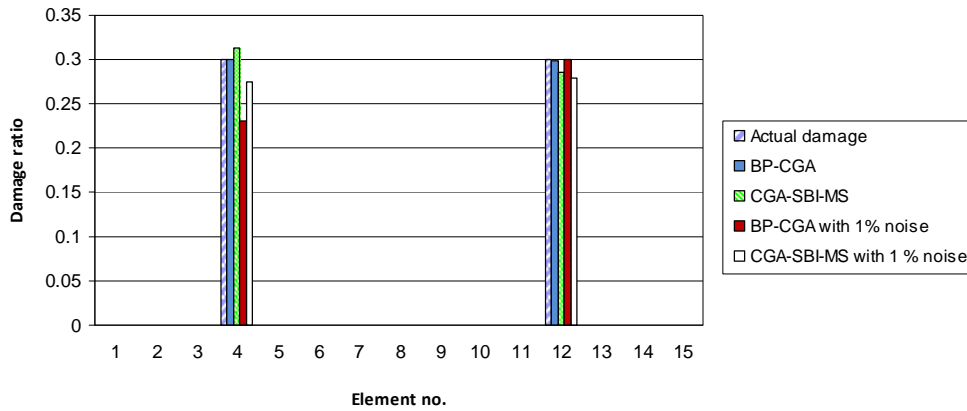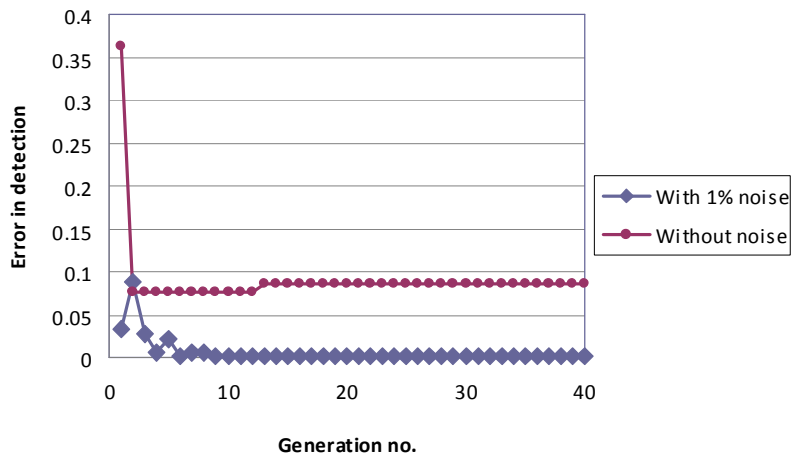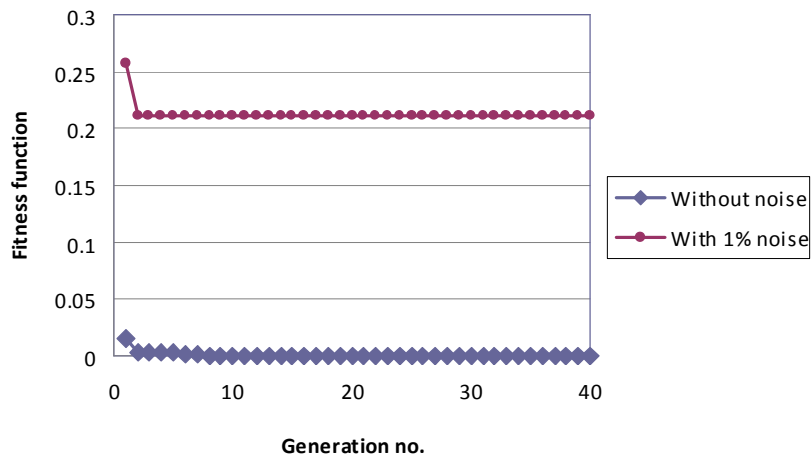(a)   (b)

Figure 9. The first stage results for 15-element prestressed concrete beam (a) without noise (b) with 1% noise

(a)

(b)

(c)

Figure 10. Solution results of BP-CGA for 15-element prestressed concrete beam: (a) damage identification results (b) convergence history of error in detection (c) convergence history of fitness function

Figure 10(a) shows that BP-CGA detects the damage locations and extents in 1% noise level with only a small error. Figures 10(b) and (c) represent the corresponding convergence history diagrams.

6.4. Runtime

For a damage detection algorithm, in addition to accuracy the rapidity is also important. To show BP-CGA is fast enough, its invested time is compared with that of CGA-SBI-MS for all three previous case studies. The results are given in Table 6. For BP-CGA the runtime in BP part (generation of the first population) and CGA part are given separately. As seen, the runtime of the proposed algorithm is significantly less than that of CGA-SBI-MS.

Table 6. The runtime of 15-element prestressed concrete beam

Algorithm	Runtime (sec)		
	Case study one	Case study two	Case study three
BP-CGA (BP + CGA = Total time)	2.41+4.02=6.43	6.46+7.03=13.49	1.83+2.99=4.82
CGA-SBI-MS (Total time)	16.41	43.04	117.06

7. CONCLUSION

Damage detection can be formulated as a nonlinear system of equations. In the case of using modal data, the number of unknowns of this system of equations is more than the number of equations. Thus there are infinite solutions which equalize the responses of damaged structure with those of analytical model. Since in the damaged cases most of the elements are healthy, the true damage solution has high sparsity. In this paper basis pursuit and continuous genetic algorithm are hybridized for damage detection. Basis pursuit is used to produce some sparse solutions for the linearized version of damage detection equations. The solutions are considered as the initial population for the continuous genetic algorithm. This initial population helps genetic algorithm to converge to the true damage solution. The proposed method is applied to a cantilever beam, a truss and a prestessed concrete beam. In the case studies both states of noisy and noise-free response data are considered. The numerical results represent that the method is both accurate and fast comparing to the methods found in the literature.

APPENDIX I

In the prestressed concrete beam, the stiffness matrix of the elements, $\mathbf{K}_e$, is evaluated as follows:

$$\mathbf{K}_e = \mathbf{K}_e^i - \mathbf{K}_e^G \qquad (I)$$

where $\mathbf{K}_e^i$ and $\mathbf{K}_e^G$ are respectively the standard and geometric stiffness matrices of the beam

which are as below:

$$\mathbf{K}_e^G = \frac{P}{30l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \text{ and } \mathbf{K}_e^i = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}, \quad \text{(II)}$$

where P is the axial force of prestressed concrete beam and l is the length of the beam element.

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