OPTIMUM DESIGN OF SKELETAL STRUCTURES USING ANT LION OPTIMIZER

S. Talatahari[1*, †]
Department of Civil Engineering, University of Tabriz, Tabriz, Iran

ABSTRACT

This paper utilizes recent optimization algorithm called Ant Lion Optimizer (ALO) for optimal design of skeletal structures. The ALO is based on the hunting mechanism of Antlions in nature. The random walk of ants, building traps, entrapment of ants in traps, catching preys, and re-building traps are main steps for this algorithm. The new algorithm is examined by designing three truss and frame design optimization problems and its performance is further compared with various classical and advanced algorithms.

Keywords: Ant lion optimizer, meta-heuristic, optimum design, truss structures; frame structures.

1. INTRODUCTION

The optimum design of structures contains minimizing the volume or weight of the structure under certain design criteria obtained by the utilized code. Until now, many different algorithms have been applied for solving this kind of problem [1]. In other words, the optimization of structures can be considered as a benchmark problem to evaluate the performance of optimization methods. The charged system search (CSS) [2], magnetic charged system search (MCSS) [3], particle swarm optimization (PSO) [4], democratic PSO (DPSO) [5], imperialist competitive algorithm (ICA) [6], colliding-bodies optimization (CBO) [7], and chaotic swarming of particles (CSP) [8] are some examples of numerous algorithms applied in this field.

Recently, Mirjalili [9] proposed a new optimization method, so-called ant lion optimizer (ALO). The hunting behaviour of antlions and entrapment of ants in antlions' traps were the main inspirations for this algorithm. Several operators were proposed and mathematically

---

[*]Corresponding author: Department of Civil Engineering, University of Tabriz, Tabriz, Iran
[†]E-mail address: siamak.talat@gmail.com (S. Talatahari)

modelled for equipping the ALO algorithm with high exploration and exploitation [9]. Five main steps of hunting prey such as the random walk of ants, building traps, entrapment of ants in traps, catching preys, and re-building traps are implemented in this algorithm. In this study, a procedure for employing an ALO method is developed for optimum design of truss structures. In order to fulfill this aim, three benchmark truss and frame structures are considered. The comparing between different results obtained by the other algorithms with the new algorithm shows advantages and disadvantages of the new method in solving structures.

2. PROBLEM STATEMENT

For optimum design of structures, the objective function can be expressed as:

$$\text{minimize } W(A) = \sum_{i=1}^{NM} \gamma_i A_i L_i \tag{1}$$

where $W(A)$ is the weight of the structure; NM is the number of members making up the structure; γ_i represents the material density of member i; L_i is the length of member i; A_i is the cross-sectional area of member i chosen between A_{min} and A_{max}; and min is the lower bound and max is the upper bound. This minimum design also has to satisfy inequality constraints that limit design variable sizes and structural responses.

Generally, for a truss structure three kinds of constraints are considered as follows:

Stress constraints: for each member the positive stress should be less than allowable tensile stress (σ_{max}) and the positive stress should be less than allowable compressive stress (σ_{min}). In each truss optimization problem, we have 2n stress constrains. These constraints can be formulated as follow:

$$\sigma_{i,\min} \le \sigma_i \le \sigma_{i,\max} ; i = 1,2,..., NM \tag{2}$$

Deflection constraints: The nodal deflections should also be limited to the maximum deflection value (δ_{max}). When a truss has NN nodes, it can be defined as:

$$\delta_j \le \delta_{j,\max} ; j = 1,2,..., NN \tag{3}$$

Buckling constraints: When a member is in compression, the buckling of the member should be controlled using the allowable buckling stress (σ_b). When NC denotes the number of compression elements, it can be formulated as:

$$\sigma_{k,b} \le \sigma_k \le 0; k = 1,2,..., NC \tag{4}$$

The constraints for frame structures are as:

Maximum lateral displacement: the maximum lateral displacement, Δ_T, should be limited as:

$$\frac{\Delta_T}{H} \leq R \tag{5}$$

where H is the height of the frame structure; R is the maximum drift index.

Inter-story displacement constraints: Similar to the previous constraint, there are some limitations for inter-story drift, Δ_j, as:

$$\frac{\Delta_j}{h_j} \leq R_I, \quad j = 1,2,...,ns \tag{6}$$

where h_j is the story height of the jth floor; ns is the total number of stories; and R_I is the inter-story drift index permitted by the code of the practice.

Stress constraints: According to AISC ASD, [10], we have:

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1, \quad \text{For} \quad \frac{f_a}{F_a} \leq 0.15 \tag{7}$$

$$\frac{f_a}{F_a} + \frac{C_{mx}f_{bx}}{\left(1 - \frac{f_a}{F_{ex}'}\right)F_{bx}} + \frac{C_{my}f_{by}}{\left(1 - \frac{f_a}{F_{ey}'}\right)F_{by}} \leq 1, \quad \text{For} \quad \frac{f_a}{F_a} > 0.15 \tag{8}$$

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1, \quad \text{For} \quad \frac{f_a}{F_a} > 0.15 \tag{9}$$

where f_a ($=P/A_i$) represents the computed axial stress. The computed flexural stresses due to bending of the member about its major (x) and minor (y) principal axes are denoted by f_{bx} and f_{by}, respectively. F_{ex}' and F_{ey}' denote the Euler stresses about principal axes of the member that are divided by a factor of safety of 23/12. The allowable bending compressive stresses about major and minor axes are designated by F_{bx} and F_{by}. C_{mx} and C_{my} are the reduction factors, introduced to counterbalance overestimation of the effect of secondary moments by the amplification factors $(1 - f_a / F_{ex}')$. For unbraced frame members, these factors are taken as 0.85. F_a stands for the allowable axial stress under axial compression force alone, and is calculated depending on elastic or inelastic bucking failure mode of the member according to the slenderness ratio.

3. ANT LION OPTIMIZER

The names of antlions originate from their unique hunting behaviour and their favourite prey. An antlion larvae digs a cone-shaped pit in sand by moving along a circular path and

throwing out sands with its massive jaw [11,12]. After digging the trap, the larvae hides underneath the bottom of the cone (as a sit-and-wait predator) and waits for insects (preferably ant) to be trapped in the pit [13]. The edge of the cone is sharp enough for insects to fall to the bottom of the trap easily. Once the antlion realizes that a prey is in the trap, it tries to catch it. When a prey is caught into the jaw, it is pulled under the soil and consumed. After consuming the prey, antlions throw the leftovers outside the pit and amend the pit for the next hunt. Another interesting behaviour that has been observed in life style of antlions is that they tend to dig out larger traps as they become more hungry and/or when the moon is full [14]. They have been evolved and adapted this way to improve their chance of survival. The main inspiration of the ALO algorithm comes from the foraging behaviour of antlion's larvae.

The ALO algorithm mimics interaction between antlions and ants in the trap. To model such interactions, ants are required to move over the search space, and antlions are allowed to hunt them and become fitter using traps. Since ants move stochastically in nature when searching for food, a random walk is chosen for modelling ants' movement as follows:

$$X(iter) = [0, cumsum(2r(1)-1), cumsum(2r(2)-1),..., cumsum(2r(n)-1)] \qquad (10)$$

where *cumsum* calculates the cumulative sum, n is the maximum number of iteration, *iter* shows the iteration of random walk, and r(t) is a stochastic function defined as follows:

$$r(t) = \begin{cases} 1 \; if \; rand > 0.5 \\ 0 \; f \; rand \leq 0.5 \end{cases} \qquad (11)$$

The position of ants and their related objective functions are saved in the matrices M_{Ant} and M_{OA}, respectively.

In addition to ants, it is assumed that the antlions are also hiding somewhere in the search space. In order to save their positions and fitness values, the $M_{Antlion}$ and M_{OAl} matrices are utilized. The pseudo codes the ALO algorithm are defined as follows, [9]:

Step 1: Initialize the first population of ants and antlions randomly. Calculate the fitness of ants and antlions.

Step 2: Find the best antlions and assume it as the elite. In this study the best antlion obtained so far in each iteration is saved and considered as an elite.

Step 3: For each ant, select an antlion using Roulette wheel and

3.1 Create a random walk using Eq. (5)

3.2 Normalize them in order to keep the random walks inside the search space

$$X_i^{iter+1} = \frac{(X_i^{iter} - a_i) \times (d_i - a_i)}{(b_i - a_i)} + c_i \qquad (12)$$

where a_i is the minimum of random walk of *i*-th variable, b_i is the maximum of random

walk in i-th variable, c_i and d_i are the minimum and maximum of i-th variable at the current iteration.

3.3 Update the position of ant

$$Ant_i^{iter+1} = \frac{R_A^{iter} + R_E^{iter}}{2} \qquad (13)$$

where R_A^{iter} is the random walk around the antlion selected by the roulette wheel; R_E^{iter} is the random walk around the elite and Ant_i^{iter+1} indicates the position of i-th ant at the iteration *iter*+1.

3.4 Update c and d using the following equations

$$c^{iter+1} = \frac{c^{iter}}{I} \qquad (14)$$

$$d^{iter+1} = \frac{d^{iter}}{I} \qquad (15)$$

where:

$$I = 10^w \frac{iter}{n} \qquad (16)$$

And w is a constant defined based on the current iteration ($w = 2$ when *iter* > 0.1n, $w = 3$ when *iter* > 0.5n, $w = 4$ when *iter* > 0.75n, $w = 5$ when *iter* > 0.9n, and w = 6 when *iter* > 0.95n).

Step 4: Calculate the fitness of all ants.

Step 5: Replace an antlion with its corresponding ant it if becomes fitter.

Step 6: Update elite if an antlion becomes fitter than the elite.

Step 7: Repeat from step 3 until a stopping criteria is satisfied.

4. DESIGN EXAMPLES

Here, three different truss and frame structures are optimized utilizing the ALO method. Then, the results are compared to the solutions of other methods to demonstrate the efficiency of the proposed method. The codes are prepared in MATLAB™ 7 and all the runs for the truss problems are implemented on a Pentium IV PC with 3.0 GHz CPU and 4 GB RAM. Due to the randomness of the algorithm, their performance cannot be judged by

the result of a single run. Many trials with independent population initializations should be
made to obtain a useful conclusion of the performance of the approach. The best, worst and
mean obtained in 20 trials are used to evaluate the performances of the algorithm.

4.1 A 72-bar space truss

For the 72-bar spatial truss structure shown in Fig. 1, the material density is 2767.990 kg/m^3
and the modulus of elasticity is 68,950 MPa. The members are subjected to the stress limits of
±172.375 MPa. The uppermost nodes are subjected to the displacement limits of ±0.635 cm in
both the *x* and *y* directions. The 72 structural members of this spatial truss are sorted into 16
groups using symmetry: (1) A1~A4, (2) A5~A12, (3) A13~A16, (4) A17~A18, (5) A19~A22,
(6) A23~A30, (7) A31~A34, (8) A35~A36, (9) A37~A40, (10) A41~A48, (11) A49~A52,
(12) A53~A54, (13) A55~A58, (14) A59~A66, (15) A67~ A70, (16) A71~A72. Table 1 lists
the values and directions of the two load cases applied to the 72-bar spatial truss.
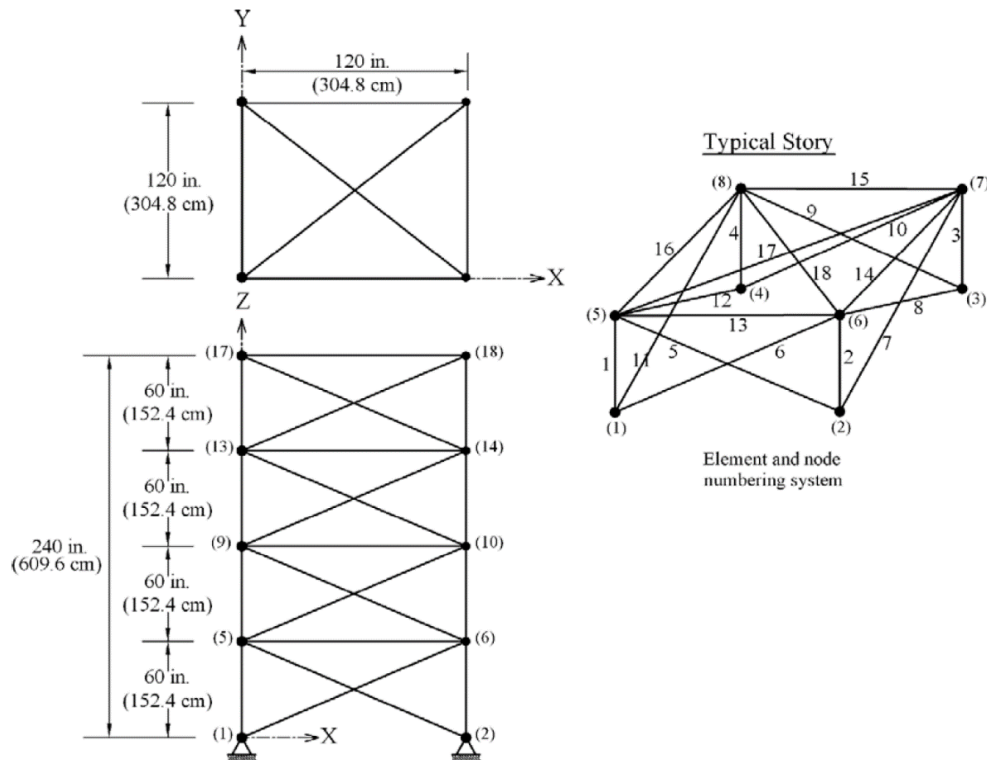

Figure 1. A 72-bar space truss

Table 2 presents the statistical results of the proposed algorithm and Fig. 2 shows the
convergence tendency of the method for the 72-bar truss problem. Table 3 compares the
results obtained by the ALO with those reported in the literature. It can be observed from
Table 2 that the design obtained by the ALO method is better than the results obtained by
many other methods.

Table 1: Loading conditions for the 72-bar spatial truss

	Case 1			Case 2		
Node	P_X kips(kN)	P_Y kips(kN)	P_Z kips(kN)	P_X	P_Y	P_Z kips(kN)
17	5.0 22.25)	5.0 (22.25)	−5.0 (22.25)	0.0	0.0	−5.0 (22.25)
18	0.0	0	0	0.0	0.0	−5.0 (22.25)
19	0.0	0.0	0.0	0.0	0.0	−5.0 (22.25)
20	0.0	0.0	0.0	0.0	0.0	−5.0 (22.25)

Table 2: Statistical results of the 72-bar truss obtained by the ALO method

Best	Mean	Worst	Std. Dev.
1,618.62	1,756.3	1,795.6	88.25
No. Antlions	No. Iterations	No. Analyses	Ave. Time (Sec.)
25	1,000	25,000	550.16

Table 3: Comparison of optimal designs for the 72-bar spatial truss structure

	Optimal cross-sectional areas (cm^2)		
Element group	PSO [15]	MSPSO [15]	ALO
A_1~A_4	12.30	12.26	12.185
A_5~A_{12}	3.43	3.26	3.342
A_{13}~A_{16}	0.064	0.064	0.0647
A_{17}~A_{18}	0.064	0.064	0.0645
A_{19}~A_{22}	8.49	8.33	8.304
A_{23}~A_{30}	3.33	3.33	3.330
A_{31}~A_{34}	0.64	0.064	0.0645
A_{35}~A_{36}	0.64	0.064	0.0655
A_{37}~A_{40}	3.45	3.34	3.408
A_{41}~A_{48}	3.26	3.35	3.366
A_{49}~A_{52}	0.07	0.07	0.0653
A_{53}~A_{54}	0.68	0.75	0.854
A_{55}~A_{58}	1.08	1.07	1.063
A_{59}~A_{66}	3.43	3.53	3.417
A_{67}~A_{70}	2.85	2.86	2.872
A_{71}~A_{72}	3.61	3.62	3.641
Weight (N)	1619.07	1618.63	1618.62

Table 4: Loading conditions for the 200 bar truss

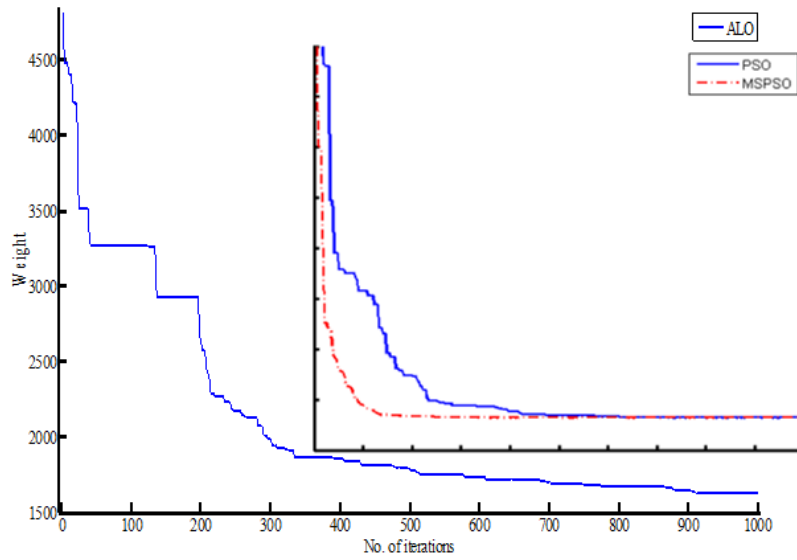| Case No.	Load (lb)	Direction	Nodes
1	1000	X	1, 6, 15, 20, 29, 34, 43, 48, 57, 62, 71
2	10,000	Y	1~6, 8, 10, 12, 14~20, 22, 24, 26, 28~34, 36, 38, 40, 42~48, 50, 52, 54, 56~62, 64, 66,68, 70~75
3			Load cases 1 and 2 acting together.

Figure 2. Convergence history for the 72- bar truss obtained by the PSO, MPSO [15] and ALO
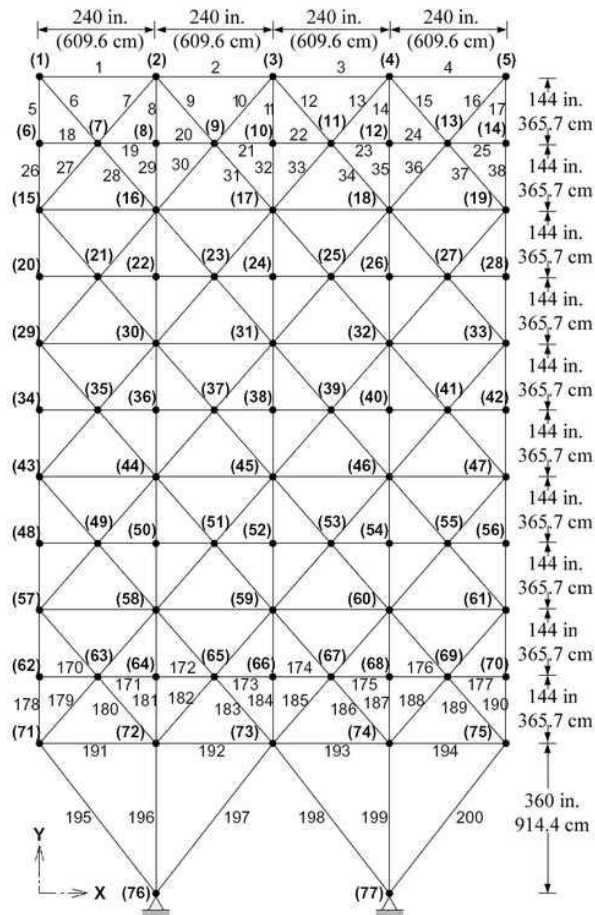

Figure 3. A 200-bar spatial truss

4.2 A 200-bar planer truss

For the 200-bar planer truss structure shown in Fig. 3, the material density is 7833.413 kg/m^3 and the modulus of elasticity is 206.91 GPa. The members are subjected to the stress limits of ±68.97 MPa. The 200 structural members of this planer truss are categorized as 29 groups using symmetry. The values and directions of the load cases applied to the 200-bar planer truss are shown in Table 4.

Table 5: Best solution results for the 200-bar truss obtained by ALO method

Best	Mean	Worst	Std. Dev.
11,538.09	12,403.25	12,998.26	822.36
No. Antlions	No. Iterations	No. Analyses	Ave. Time (Sec.)
50	1,000	50,000	802.36

The minimum weight and the statistical values of the best solution obtained by the new method are presented in Table 5. Table 6 compares the results of the ALO to those of the previously reported methods in the literature. In this table, the results of some well-known methods such as SA [16], centers and force formulation (CP) [17], augmented Lagrangian methods (AL) [18], GA [19] and a genetic-nelder mead simplex algorithm (GNMS) [20] are presented. As shown in the this table, the proposed algorithm can find the best design, among the other existing studies and the best weight of the ALO algorithm is 11538.09 kg.
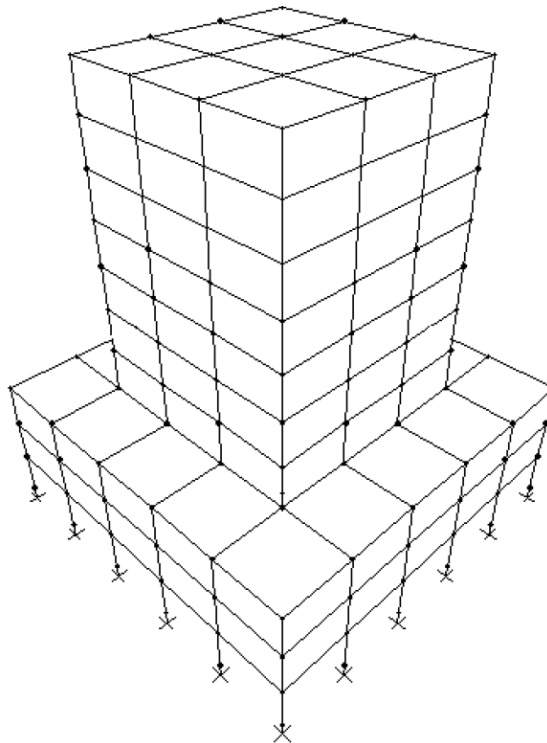
Figur 4. A 10-story space frame

Table 6: Optimum designs for the 200-bar truss problem

	Optimal cross-sectional areas (cm^2)					
Element group	SA [16]	CP [17]	AL [18]	GA [19]	GNMS [20]	ALO
$A_{1\sim4}$	0.1468	0.147	0.148	0.347	0.147	0.142
$A_{5, 8, 11, 14, 17}$	0.94	0.945	0.945	1.081	0.935	0.953
$A_{19\sim24}$	0.100	0.100	0.100	0.100	0.105	0.100
$A_{18, 25, 56, 63, 94, 101, 132, 139, 170, 177}$	0.100	0.100	0.100	0.100	0.106	0.100
$A_{26, 29, 32, 35, 38}$	1.940	1.945	1.945	2.142	1.944	1.932
$A_{6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37}$	0.296	0.297	0.298	0.347	0.298	0.294
$A_{39\sim42}$	0.100	0.100	0.100	0.100	0.100	0.108
$A_{43, 46, 49, 52, 55}$	3.104	3.106	3.123	3.565	3.106	3.125
$A_{57\sim62}$	0.100	0.100	0.100	0.347	0.109	0.100
$A_{64, 67, 70, 73, 76}$	4.104	4.105	4.123	4.805	4.109	4.121
$A_{44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75}$	0.403	0.404	0.399	0.440	0.403	0.399
$A_{77\sim80}$	0.191	0.193	0.100	0.440	0.196	0.107
$A_{81, 84, 87, 90, 93}$	5.428	5.429	5.393	5.952	5.420	5.398
$A_{95\sim100}$	0.100	0.100	0.100	0.347	0.100	0.100
$A_{102, 105, 108, 111, 114}$	6.428	6.429	6.393	6.572	6.425	6.387
$A_{82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113}$	0.573	0.575	0.526	0.954	0.577	0.526
$A_{115\sim118}$	0.133	0.134	0.435	0.347	0.133	0.430
$A_{119, 122, 125, 128, 131}$	7.972	7.974	7.95	8.525	7.978	7.935
$A_{133\sim138}$	0.100	0.100	0.100	0.100	0.100	0.100
$A_{140, 143, 146, 149, 152}$	8.972	8.974	8.95	9.300	8.963	8.946
$A_{120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151}$	0.705	0.705	0.859	0.954	0.700	0.853
$A_{135\sim156}$	0.420	0.422	0.150	1.764	0.425	0.156
$A_{157, 160, 163, 166, 169}$	10.864	10.868	10.998	13.300	10.859	10.995
$A_{171\sim176}$	0.100	0.100	0.100	0.347	0.100	0.100
$A_{178, 181, 184, 187, 190}$	11.861	11.867	11.998	13.3	11.864	11.983
$A_{158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189}$	1.0339	1.0349	0.913	2.142	1.029	0.921
$A_{191\sim194}$	6.6818	6.685	6.662	4.805	6.680	6.668
$A_{195, 197, 198, 200}$	10.811	10.810	10.806	9.300	10.816	10.804
$A_{196, 199}$	13.840	13.838	13.824	17.170	13.829	13.808
Weight (kg)	11542.14	11,557.29	11,542.46	12,958.98	11,553.97	11,538.09

4.3 A 10-story space frame

A 10-story space steel frame consisting of 256 joints and 568 members is considered as shown in Fig. 4. The detailed information about the grouping of elements and loading conditions are presented by Saka and Hasançebi [21].

The optimum design of this space frame is carried out using the simulated annealing (SA), evolution strategies (ESs), particle swarm optimizer (PSO), tabu search optimization (TSO), simple genetic algorithm (SGA), ant colony optimization (ACO), harmony search (HS) methods and the new ALO method. In each optimization technique the number of iterations has been taken as 50,000 for all methods. The design history of hybrid algorithm is shown in Fig. 5. The optimum design attained by the ASLO for this example is 226,056.3 kg, while it is 232,301.2 kg, 228,588.3 kg for the AHS and ESs which are the best ones among the others. The minimum weights as well as W-section designations obtained by the TSO, ESs, AHS and the new algorithms are provided in Table 7. For the present algorithm, maximum stress ratio is equal to 96.80%, and the maximum drift is 0.89 cm, while the allowable value is set to 0.9144 cm.

Table 7:Optimal design for the 10-story space frame

| Element group	Optimal W-shaped sections			
	TSO [21]	AHS [21]	ESs [21]	ALO
1	W14X193	W14X176	W14X193	W36X150
2	W8X48	W14X48	W8X48	W24X62
3	W8X40	W10X39	W10X39	W10X39
4	W10X22	W10X22	W10X22	W14X26
5	W21X50	W24X55	W21X50	W21X44
6	W10X54	W12X65	W10X54	W21X62
7	W14X120	W14X145	W14X109	W24X104
8	W14X159	W14X159	W14X176	W14X159
9	W21X44	W14X30	W18X40	W21X50
10	W18X40	W18X40	W18X40	W12X45
11	W10X45	W10X54	W10X49	W12X58
12	W14X90	W14X90	W14X90	W18X86
13	W12X120	W14X120	W14X109	W14X109
14	W12X44	W14X34	W14X30	W18X40
15	W16X36	W18X40	W16X36	W18X40
16	W10X33	W8X31	W12X45	W16X77
17	W12X65	W12X65	W12X65	W12X50
18	W14X34	W18X35	W10X22	W12X72
19	W12X79	W12X79	W12X79	W16X36
20	W14X30	W14X30	W14X30	W10X33
21	W10X39	W10X22	W8X35	W8X28
22	W12X45	W10X45	W10X39	W8X24
23	W12X35	W8X31	W8X31	W16X31
24	W6X20	W10X22	W8X18	W8X24
25	W12X26	W12X26	W14X30	W10X33
Weight (kg)	235,167.5	232,301.2	228,588.3	226,056.3
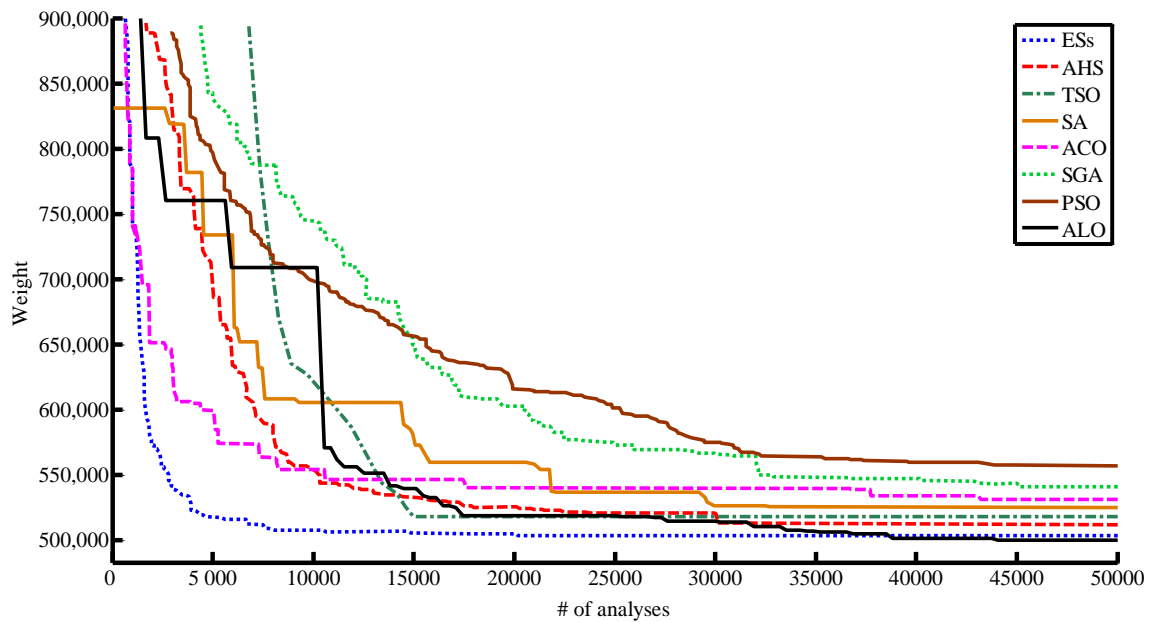
Figure 5. Convergence history for the 10-story space frame

6. CONCLUDING REMARKS

The hunting behaviour of antlions and entrapment of ants in antlions' traps were the main inspirations for the ALO algorithm. Five main steps of hunting prey such as the random walk of ants, building traps, entrapment of ants in traps, catching preys, and re-building traps are implemented in this algorithm. In this paper, the ALO algorithm is investigated on solving skeletal structures. Three numerical examples are solved by the ALO algorithm and its performance is further compared with various classical and advanced algorithms. From results presented in this study, the ALO algorithm shows a good performance compared to some other well-known meta-heuristics such as GA, SA, PSO, MPSO, CP, GNMS, ESs, TSO, ACO, and HS techniques. The next work should focus on improving this algorithm for large-scale problems where the performance of the algorithm is not good as its ability on solving small ones. For examples, for the 200-bar truss, after almost 10,000 analyses the ALO could not determine the optimum domain and the so far best results in this iteration is almost 3 times larger than the final optimum point. Applying this method to some other discrete problems can be another field of research. Clearly for this aim, some modifications and improvisations are necessary.

REFERENCES

1. Kaveh A. *Advances in Metaheuristic Algorithms for Optimal Design of Structures*, Springer International Publishing, Switzerland, 2014.
2. Kaveh A, Talatahari S. A novel heuristic optimization method: charged system search, *Acta*

Mech 2010; **213**(3-4): 267-89.

3. Kaveh A, Motie Share MA, Moslehi M. A new meta-heuristic algorithm for optimization: magnetic charged system search, *Acta Mech* 2013; **224**: 85-107.

4. Gomes HM. Truss optimization with dynamic constraints using a particle swarm algorithm, *Expert Sys Appl* 2011; **38**(1): 957-68.

5. Kaveh A, Zolghadr A. Democratic PSO for truss layout and size optimization with frequency constraints, *Comput Struct* 2014; **130**: 10-21.

6. Talatahari S, Kaveh A, Sheikholeslami R. Chaotic imperialist competitive algorithm for optimum design of truss structures, *Struct Multidiscip Optim* 2012; **46**(3): 355-67.

7. Kaveh A, Mahdavi VR. *Colliding Bodies Optimization: Extensions and Applications*, Springer Verlag, Berlin, New York, 2015.

8. Kaveh A, Sheikholeslami R, Talatahari S, Keshvari M. Chaotic swarming of particles: a new method for size optimization of truss structures, *Adv Eng Softw* 2014 **67**: 136-47.

9. Mirjalili S. The ant lion optimizer, *Adv Eng Softw* 2015; **83**: 80-98.

10. *American Institute of Steel Construction (AISC). Manual of Steel Construction-Allowable Stress Design*, 9th ed, Chicago, IL, 1989.

11. Scharf I, Subach A, Ovadia O. Foraging behaviour and habitat selection in pitbuilding antlion larvae in constant light or dark conditions, *Anim Behav* 2008; **76**: 2049-57.

12. Griffiths D. Pit construction by ant-lion larvae: a cost-benefit analysis, *J Anim Ecol* 1986; **55**: 39-57.

13. Scharf I, Ovadia O. Factors influencing site abandonment and site selection in a sit-and-wait predator: a review of pit-building antlion larvae, *J Insect Behav* 2006; **19**: 197-218.

14. Grzimek B, Schlager N, Olendorf D, McDade MC. *Grzimek's Animal Life Encyclopedia*, Michigan: Gale Farmington Hills; 2004.

15. Talatahari S, Kheirollahi M, Farahmandpour C, Gandomi AH, A multi-stage particle swarm for optimum design of truss structures, *Neural Comput Appl* 2013; **23**(5): 1297-1309.

16. Lamberti L. An efficient simulated annealing algorithm for design optimization of truss structures, *Comput Struct* 2008; **86**: 1936-53.

17. Farshi B, Alinia-ziazi A. Sizing optimization of truss structures by method of centers and force formulation, *Int J Solids Struct* 2010; **47**: 2508-24.

18. Coster IE, Stander N. Structural optimization using augmented Lagrangian methods with secant Hessian updating, *Struct Opitim* 1996; **12**: 113-19.

19. Togan V, Daloglu AT. An improved genetic algorithm with initial population strategy and self-adaptive member grouping, *Comput Struct* 2008; **86**: 1204-18.

20. Rahami H, Kaveh A, Aslani M, Najian Asl R. A hybrid modified genetic-nelder mead simplex algorithm for large-scale truss optimization, *Int J Optim Civil Eng* 2011; **1**: 29-46.

21. Saka MP, Hasançebi O. Design code optimization of steel structures using adaptive harmony search algorithm, Chapter 3 of a book entitled: *Harmony Search Algorithms for Structural Design*, Edited by ZW Geem, Springer Verlag, 2009.